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Constructing competence: an analysis of student participation in the activity systems of mathematics classrooms

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Abstract This paper investigates the construction of systems of competence in two middle school mathematics classrooms. Drawing on analyses of discourse from videotaped classroom sessions, this paper documents the ways that *agency* and *accountability* were distributed in the classrooms through interactions between the teachers and students as they worked on mathematical content. In doing so, we problematize the assumption that competencies are simply attributes of individuals that can be externally defined. Instead, we propose a concept of individual competence as an attribute of a person's participation in an activity system such as a classroom. In this perspective, what counts as "competent" gets constructed in particular classrooms, and can therefore look very different from setting to setting. The implications of the ways that competence can be defined are discussed in terms of future research and equitable learning outcomes.

Keywords Classroom culture · Discourse analysis · Competence · Middle school · Urban schools

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1 Introduction

The purpose of this paper is to offer a framework for considering the process of competence construction in mathematics classrooms. In doing so, we seek to highlight the constructed nature of competence, in order to problematize the customary usage of the term in discussions of learning and teaching, and in particular in American state and national standards documents. In these discussions, competence refers to a collection of skills or abilities that are attributed to individuals apart from the specific contexts in which they participate. For example, we talk about the mathematical competencies that we want all students to acquire, or the level or proficiency that we hope students will attain (Kilpatrick, Swafford and Findell 2001; National Council of Teachers of Mathematics 2000; National Mathematics Advisory Council 2008; Schoenfeld 1999). There is reason to worry about the content that students engage in their classrooms, and we do not want to brush those concerns aside. The aim of this paper, however, is to problematize the assumption that competencies are simply attributes of individuals. Instead, we propose a concept of competence as an attribute of participation in an activity system (Engeström 1993; Greeno 2006; Lemke 1990) such as a classroom. In this perspective, what counts as "competent" gets constructed in particular classrooms, and can therefore look very different from setting to setting. Viewing competence as constructed has important implications for both theory and practice, as it requires shifting the focus of analysis from the individual to the classroom in which the individual is a participant. As a consequence, whether or not a student is deemed to be "competent" is no longer seen as a trait of that student, but rather an interaction between the opportunities that a student has to participate competently and the ways that individual takes up those opportunities. Studying the classroom as an "activity system," which we define as a social organization that may contain learners, teachers, curriculum materials, software tools, and the physical environment, is appropriate for understanding how all these aspects interact in the construction of competence.

This work contributes to a body of literature that has sought to establish that classrooms are social systems that are organized through regularities of shared practice (Bauersfeld 1992; Brown 1994; Cobb 1999, 2000; Yackel and Cobb 1996). In particular, these systems of meaning shape the ways that individuals are expected, entitled, and obligated to participate, but also the meanings that members make of particular acts of participation (Gee 1999; Holland, Skinner, Lachicotte and Cain 1998; Jungwirth 1991). In this paper, we address a specific aspect of the classroom system—the ways that competence is constructed amongst participants in particular classroom contexts. This focus is important because it enables an illustration of the ways that students' participation—and in particular, attributions of participation—truly are a function of the relationship between what participants do given what they have *opportunities* to do (Gresalfi 2004). More specifically, we focus on two aspects of classrooms that support the ways that systems of competence are constructed—the ways that agency and accountability are distributed in the classroom. By highlighting these two aspects of classroom participation structures in particular, we seek to provide both an analytic lens for future research, and a tool for practitioners as a means of better understanding the complexities of classroom systems.

1.1 Competence, redefined

Competence, as we use the term, refers generally to what students need to know or do in order to be considered successful by the teacher and other students in the classroom. This is related to the notion of *normative identity,* as discussed by Cobb, Gresalfi and Hodge

(2008). Both terms refer to the system of expectations that is established around accepted mathematical practice. In this paper we focus on competence in an effort to highlight not just the kinds of practices that are considered normative (accepted aspects of collective activity), but those activities that are specifically associated with designations of success. In examining classroom systems at this level, we seek to demonstrate a method for better understanding the mechanisms underlying the ways that normative constructions of competence, or normative identities more broadly, become established.

One version of competence involves what students need to know or do in order to be "correct," but there can be other versions. For example, sharing mistakes or misunderstandings can be a useful learning activity, and the person who has shared a mistaken idea can be considered to be competent (Hiebert, Carpenter, Fennema, Fuson, Wearne and Murray 1997; Lampert 2001). This distinction highlights the constructed nature of competence as it is locally and individually defined through participation between the teacher and students. As a consequence, the competencies that are constructed in a classroom impact both what counts as mathematics, and who gets to say (Amit and Fried 2005; Cobb, Wood and Yackel 1993; Cobb, Wood, Yackel and McNeal 1992; Yackel and Cobb 1996). In turn, their negotiation can impact and change which behaviors are sanctioned as competent.

The process of negotiating what it means to "do mathematics," and which elements of "doing mathematics" are reinforced and highlighted does not happen in one moment, or in just one way. A behavior, way of thinking, or contribution gets positioned as competent most often by the teacher, who generally holds power in classrooms to determine what is both correct and acceptable. For example, a teacher's response to a student's contribution, such as "nice job!" quickly and clearly positions that contribution as competent[1]. It is not only correct responses that can be counted as competent, however. At times a teacher may intentionally reinforce a particular way of doing mathematics that may not be comfortable for students, such as by explicitly positioning a student's mistaken solution as an important contribution for the learning of the class (c.f. Lampert 1990, 2001). In that context, the act of sharing any solution is positioned as competent, as it serves to push the thinking of the class.

Activity systems such as classrooms are not defined by one member, no matter how powerful that member may be. Rather, they are "dynamic, open, semiotic system(s) of meaningful actions and meaning-making processes" (Lemke 1990: p. 191). Thus, the teacher is not the only participant who is able to shape the construction of competence in a classroom; students also play a role in this negotiation. In the same example above, students might thwart a teacher's attempt to support the idea that making mistakes is valuable if they mock or belittle a student who has made a mistake. Students may also play a more positive role in constructing competence, from taking up opportunities to participate in ways advocated for by the teacher, to consistently asking particular students for help or checking with particular students to make sure that their own answer is correct. Both kinds of acts—from identifying to openly resisting—contribute to the ongoing negotiation of the classroom culture (Cobb, Gresalfi and Hodge 2008).

[1] This move simultaneously serves to reinforce the relative power difference between teacher and students by indexing the teacher's position relative to the student as the person who is responsible for determining the flow of activity and the direction of the discussion (Lemke 1990). Although students can also utter the same words, the meaning of such an utterance is slightly different as it does not (usually) promise the same kinds of potential outcomes (namely, good grades). In some rare classrooms this structure can be disrupted, but it is difficult to imagine a complete reversal of these positions.

Considering competence as constructed in interaction draws attention to the differences between how competent behavior might be defined across classrooms. A student who is considered to be competent in one mathematics classroom may not necessarily be considered competent in another classroom (Horn 2008). This suggests that considerations of *who* is good at mathematics should be accompanied by questions of *why* people are good at mathematics, and more importantly, *how we know*. Although being considered to be good at mathematics requires particular kinds of performances on the parts of students, it also requires those performances are recognized and given meaning (Lemke 1990). For example, McDermott (1993) highlights the complex interaction required to give meaning to any individual's actions in his well-known essay on a child who had been diagnosed with a learning disability: "On any occasion of his looking inattentive, for example, it took Adam to look away at just the right time, but it took many others to construct the right time for Adam to look away; it took others to look away from his looking away, and still more to discover his looking away, to make something of it, to diagnose it, to document it, and to remediate it" (p. 273).

1.2 Why competence?

What is the purpose of focusing on mathematical competence? This paper is intended to contribute to the ongoing conversation about what it means to be "mathematically proficient" (National Council of Teachers of Mathematics 2000; Kilpatrick et al. 2001). There are at least two parts to this challenge: the first is to determine what is meant by *proficiency,* and the second is to determine how to support all students in becoming proficient at mathematics. Our focus on the construction of mathematical competence considers both goals by deconstructing how mathematical competence gets defined, and thus potentially supported, in interactions.

The idea that mathematical competence is constructed in classroom interaction is especially important as there is so much agitation in the mathematics education community, and in schools more broadly, about choosing the "right" curriculum, and proving that it is effective. As our analyses will illustrate, the curriculum that is used by the teacher shapes but does not by any means determine what actually happens as students begin to engage with mathematics tasks. This work also has important implications for equity. As Ladson-Billings (1997) notes, part of equitable teaching involves focusing on student thinking, and organizing instruction around what students know or are bringing to the table. As our analysis will illustrate, as systems of competence are constructed, choices are made about the range of behaviors that will be incorporated into the classroom system. This process can result in accommodating "what students need to do in order to be successful" to the students' needs, placing the students' experiences at the center of classroom decision-making. (Ladson-Billings 1997; Gutiérrez 2004).

2 A framework for conceptualizing competence as it is realized in the classroom

When we focus on aspects of activity involving competence, we refer to a *system of competence* that is constructed by the participants in their practice. This system of competence gets constructed as students and the teacher negotiate (1) the kind of mathematical agency that the task and the participation structure afford, (2) what the students are supposed to be accountable *for* doing, and (3) whom they need to be accountable *to* in order to participate successfully in the classroom activity system.

2.1 The distribution of agency

As we use the term, an individual's agency refers to the way in which he or she acts, or refrains from acting, and the way in which her or his action contributes to the joint action of the group in which he or she is participating[2]. An individual's participation in a group or community can come to develop a pattern, so that a set of expectations by her or him and by others regarding the agency of her or his actions develops[3]. The pattern of agency that characterizes an individual's participation is an important aspect of her or his identity in a group or community. It is important here to dispel the notion that people "have" or "lack" agency. In virtually any situation, even the most constrained, people are able to exercise agency; at the basic level, by complying or resisting. The *ways* that agency can be exercised, and the consequences for doing so, are what change in a particular context. Said differently, an individual can *always* exercise agency, it is the nature of that exercise that differs from context to context.

As educators, we are not only concerned with whether or not students "act," but how they are given opportunities to act. We are interested in understanding the distribution of agency—do students have opportunities to either engage in one particular way or not engage at all? Or are there multiple ways that a student might choose to productively engage in a classroom activity? Pickering (1995) focused on these kinds of distinctions when he developed the terms *conceptual* and *disciplinary* agency in his sociohistorical analysis of a case of research in mathematics. Mathematicians exercise conceptual agency when they engage in decision making, exploration, and strategizing, for example, when they choose methods and consider and develop meanings and relations of concepts and principles. When they decide to use an established method, agency is turned over to the discipline, rather than being the responsibility of the mathematician (other than carrying out the prescribed steps correctly).

2.2 Accountability to whom for what?

In our use of the term, accountability refers both to what students are supposed to know (be accountable for), and who students are expected to convince (be accountable to). The former use of the term is more typical in discussions of assessment and schooling in the United States, as there is an understandable focus on the content with which we expect students to engage. However, the latter use of the term is particularly informative in the context of considering how competence gets constructed in a classroom. Specifically, classrooms in which students are accountable for demonstrating their understanding only to the teacher require convincing only one person (who, ostensibly, already understands the mathematical content) that an explanation or solution makes sense. In contrast, classrooms

[2] A person's agency in a brief episode of interaction is, in part, whether he or she initiates an idea, agrees with, elaborates on, questions, or disagrees with what someone else initiated, or refrains from responding. It also depends on whether her or his action is accepted, elaborated, questioned, challenged, or ignored.

[3] Here, we are talking about characteristic ways of engaging in the classroom, at the level of what Cobb and his colleagues (Cobb and Yackel, 1998; Forman 1996; Yackel and Cobb, 1996) call *social norms*. Learning, as we and others conceive of it, can be seen through individual students' changes in participation with respect to particular content (c.f. Lave and Wenger 1991). Thus, although often students have characteristic ways of participating in the classroom activity system (c.f. Gresalfi 2004), their participation may change, particularly with respect to their engagement with particular ideas. These processes are intermingled, as students' participation with the social norms of the classroom (i.e. tendency to justify ideas or not) shapes the ways that she might engage with particular mathematical ideas, and vice versa.

in which students are accountable to their classmates as well as their teachers often require that students do much more work in order to convince their peers that their solutions make sense. As a consequence, students in such classrooms may have many more opportunities to respond to questions and revise their solutions.

As illustrated in Fig. 1, we submit that these aspects of mathematical competence get negotiated and defined as students and the teacher work together to establish and maintain a participation framework (Goffman 1981) in the classroom, which involves, for example, establishing what the class will consider to be a convincing argument, or deciding what the norms of argumentation will be. Likewise, what the teacher and the students take themselves to be working on gets established as they negotiate, for example, the requirements of the task for sense-making. As these aspects of classroom practice are negotiated, issues such as accountability and agency get arranged. For example, as students and the teacher negotiate who is expected, entitled, or obligated to author or critique mathematical ideas, they are simultaneously working to establish to whom students are accountable for making sure that their answers make sense. This is, of course, constantly renegotiated, and may change slightly from task to task. Likewise, the openness of a given task shapes the kinds of agency that students have opportunities to exercise as they complete the task.

In the following sections of this paper, we explore the proposed framework for documenting how systems of competence are constructed in classrooms by considering excerpts from two classrooms that were a part of a larger research project that took place in an urban middle school in the San Francisco Bay Area. Our purpose here is not to document *what it means to be competent*; although we will address that issue with respect to the two classrooms discussed. Rather, our purpose is to use these classrooms as examples to think about how competence gets made meaningful through the actions of a community (Lemke 2000). In the sections that follow, we examine two classrooms and discuss and contrast the nature of competence that was constructed. We also explore the relative flexibility of systems of competence by focusing on how designations of competent mathematical behavior can be broadened to include and honor students' mathematical experiences. We will use these examples to reflect back on our framework for analyzing the

<table>
<tr><td rowspan="6">Negotiation of</td><td rowspan="3">Participation structure</td><td>How an idea enters the common ground (i.e. standards of evidence)</td><td rowspan="6">Leads to definitions of</td><td rowspan="2">What students are accountable for</td></tr>
<tr><td>Who is expected to author or critique mathematical ideas</td></tr>
<tr><td>Norms of argumentation (how students are encouraged to talk to each other)</td><td rowspan="2">Who students are accountable to</td></tr>
<tr><td rowspan="3">Task as realized</td><td>Requirements for sense-making</td></tr>
<tr><td>Openness of the task (opportunities to use different methods, find solutions)</td><td rowspan="2">What kinds of agency students can exercise</td></tr>
<tr><td>Requirements for successful completion</td></tr>
</table>

Fig. 1 A model of how competence gets constructed in a classroom

construction of competence, and discuss the implications of this understanding for our understanding of student learning, and more specifically, student proficiency.

3 Methods

3.1 Participants

The two classrooms that we consider in this paper come from an American school which has a history of participating in a mathematics reform effort, *The Algebra Project*, founded by civil rights leader Robert Moses (Moses and Cobb 2001). The Algebra Project is a curriculum designed to enable more students to take Algebra, a course that is now required by most schools for high school graduation, and by all four-year colleges for admission. Briefly, the Algebra Project employs a “five-step curricular process” for learning mathematics, designed to help students to transition from and make connections between their everyday lives and the world of mathematics. This process includes: 1) engaging in a shared experience (such as a subway ride, bus ride, etc.); 2) modeling these events; 3) working out an *Intuitive Language Response*; 4) shifting to *Regimented English*; and finally 5) creating symbolic representations of the experience. Although not all teachers in the school were formally trained using the Algebra Project Curriculum, all sixth grade students were taught using the Algebra Project, and the spirit of the curriculum was at the forefront of discussions between teachers during department meetings.

3.2 Data collection

The excerpts that we discuss come from a larger dataset of videotapes that were collected over the 2001–2002 school year. This dataset included videotapes from each of three classrooms—one 6th grade, one 7th grade, and one 8th grade, representing half of the mathematics classrooms in this school. Each classroom was visited between ten to twelve times from January through May. Each classroom visit was documented with both observation notes and videotapes. Classes were videotaped with two cameras. One camera focused on a group of students (students in all mathematics classes were arranged in groups) for the duration of a lesson; a second camera was placed at the back of the room, in order to capture the overall action in the classroom. When students were working in groups, this camera followed the teacher's progress around the room.

We focus in particular on one sixth grade teacher and one eighth grade teacher. The sixth grade teacher had extensive experience with the Algebra Project, and had taught the curriculum many times. The eighth grade teacher taught Algebra, and was not trained specifically as an Algebra Project teacher. We have chosen to focus on these two classrooms because of the contrast they provide in illustrating the different kinds of systems of competence that can be constructed in well-functioning classrooms. Our analysis, described in detail below, was grounded in transcripts of talk and gesture, and videotapes of classroom interaction.

3.3 Analysis

Our analysis is grounded in a theoretical paradigm that considers interaction amongst participants and elements of an activity system as the unit of analysis (Bauersfeld 1995; Greeno and MMAP 1998). Our analysis considered negotiations among participants in order to document the negotiation of shared (or "taken-as-shared") meanings (Cobb and Yackel 1998; Cobb, Stephan, McClain and Gravemeijer 2001). In particular, we considered the affordances of various aspects of the classroom systems, and the ways that these affordances were taken up (or not) by various participants. We focused first on the affordances of the *mathematical tasks* that the students and the teacher were working on. These tasks created affordances for the activity of the class by structuring the kinds of mathematical knowledge that students had opportunities to use and build. They also provided frameworks for the ways that mathematical knowledge got constructed—individually, in pairs, with the entire class, and with the teacher. However, the tasks did not determine in detail what happened in the individual classrooms. Instead, the definition of a task, and the way that task was carried out, happened in interaction between the teacher and the students (c.f. Henningsen and Stein 1997; Stein, Smith, Henningsen and Silver 2000).

In our analysis, we considered two related characteristics of tasks: their *cognitive demands,* and the opportunities that were afforded for students to complete the task by exercising either *conceptual* or *disciplinary agency* (Pickering 1995). The concept of cognitive demands was developed by Stein and her colleagues in their analyses of tasks that they observed in classrooms in the QUASAR project (Stein, Smith, Henningsen and Silver 2000). They distinguished between tasks that require retrieval of a term or fact from memory, an application of a procedure without reference to its meaning, an application of a procedure with consideration of its meaning, or doing mathematics. Doing mathematics involves consideration of meanings of concepts and methods. The levels of cognitive demand of tasks discussed by Stein et al. (2000) differ in the kinds of agency Pickering (1995) distinguished. Recalling facts or definitions and executing procedures involve disciplinary agency; there are correct answers, and a student either gets it right or doesn't. Procedures with connections and, especially, doing mathematics generally involve conceptual agency, with students being positioned to take initiative in constructing meaning and understanding of the methods and concepts that are the subjects of their learning.

We also focused on the affordances offered by participants (i.e. the teacher, students) for engaging with mathematics. For example, we examined the ways that the teacher introduced and positioned the task, and the ways that the teacher engaged with students around the task as they completed it. Likewise, we considered the ways that the students spoke to each other and to the teacher as they worked through tasks, paying particular attention to whom the students believed themselves to be accountable to, and who they considered to be competent in a given moment.

Finally, we examined places where there was direct evidence that a student's act of reasoning, behavior, etc., was considered to be competent. For example, one direct piece of evidence can be seen when the teacher or another student presents validation that a student's contribution is valuable. A teacher saying: "Good job," or "You're on the right track," is an indication that the teacher approves the student's performance, and usually provides clear evidence of a student's contribution being constructed as competent[4].

Likewise, when a student's behavior is cited as being off-task or incorrect (e.g., "You're on the wrong track", or "You're behind") the students' performance is characterized as lacking in competence at the moment. It is important to note that such moments do not

[4] However, whether the inference of competence is justified by a positive evaluation of performance depends on the way in which the participants attribute causality for the valuable contribution. For example, that inference can be qualified or negated if there is evidence of a different causal attribution (e.g., "That was a lucky guess").

necessarily lead to judgments that the student lacks competence in general. The teacher or another student may indicate that he or she judges the student as capable of performing competently, and may act so as to motivate the student. For example, if a student is not performing an assigned task and the teacher declines to provide direction for the student, they (usually) implicitly construct the student as having the potential to perform competently, at least with whatever guidance the teacher may have provided. If at that point the student does not attempt to perform competently, there is a good chance that the student will be seen as being unmotivated or resistant.

We also interpret evidence of a less direct kind as supporting inferences about competence that is constructed in classroom interactions. When a teacher assigns a task and expresses her or his expectation that students will work productively on the task, the teacher's action presupposes that students are competent to do that work. When students engage in work on the task, they accept that attribution of competence, at least to some degree. Likewise, when a teacher interacts with a student who is not engaged in the task he or she has assigned, and the teacher attempts to change the student's state of mind to become engaged, we take that effort as evidence of the teacher's attribution of competence by the student to work productively on the task, if the student becomes more engaged.

3.4 Challenges in seeing competence

It is easiest to see how competence is getting constructed when aspects of the classroom are operating in alignment—that is, when affordances of an activity in terms of exercising agency are supported by other aspects of the classroom system, such as the requirements for accountability. For example, tasks that involve opportunities for students to exercise conceptual agency are likely to result in more than one solution path. Resolving these different solutions and understanding how they are related requires a system of accountability wherein students need to be able to convince someone else of the reasonableness of the method they chose. Alignment makes it easier to see what is getting counted as competent because there are multiple aspects of the classroom system that are working together to focus on determining ways of participating as competent. In contrast, when aspects of the classroom are affording different, sometimes conflicting ways of participating, such as when the task is set up by the teacher as allowing for multiple solution paths (conceptual agency), but the teacher appears to be looking only for one (accountability to the teacher, disciplinary agency), it is harder to determine what ultimately is getting constructed as competent.

Another kind of alignment that makes systems of competence easier to see appears when there is alignment within the classroom between the teacher, students, and tasks—when the teacher's expectations are met by the students, and vice versa, and when the affordances of the activity are aligned with the accomplishment of that activity by both the teacher and the students. Misalignments occur around issues of competence that are contested, often with students resisting the norms of participation and accountability that a teacher or other students are attempting to apply. For example, in one classroom, the teacher and students might be working together on a task designed to help students learn the rule for dividing fractions. We can see competence getting done in this classroom vis-à-vis “learning the rule” when things are running smoothly—when the students expect, and the teacher provides, help along these lines. We can also see evidence of this being the activity when there is a breakdown, and things don't run smoothly—when the teacher doesn't provide the feedback that the students anticipate, or the students don't work along the lines that the teacher expects.

As we will illustrate in the analyses that follow, the activity in two classrooms (6th grade and 8th grade) seems to be operating quite fluidly, as the activity, teacher, and students seem to be well-aligned—what counted as math, and who got to say, ended up being inseparable from each other. However, the particulars of these interactions (specifically *what* counted as math, and *who* got to say), looked quite different from place to place.

4 Systems of competence: Examples from classrooms

4.1 Working hard and fixing mistakes: Two episodes in a sixth grade classroom

Our first examples come from a sixth-grade classroom in which students were working on a relatively open-ended task of creating and interpreting symbols. This activity positioned students with *conceptual agency* for creating sensible symbols, and *accountability for* making symbols that sensibly communicated information to others. Likewise, they were *accountable to* each other for determining the interpretability of those symbols.

As the students were positioned with respect to agency and accountability, the teacher and the students constructed a system of competence that identified "doing math" as a process of production, evaluation, and revision. Furthermore, it was repeatedly emphasized that engaging in this process was challenging and sometimes difficult. Thus, in this classroom the system of competence that was established was not about following set directions or being able to finish quickly. Rather, what counted as competent here involved considering the sensibility of mathematical decisions and negotiating with others about the meaning of mathematical representations, a process that was both challenging and time-consuming. To be competent, students needed to engage in behaviors such as 1) noting when something did not make sense, 2) making that confusion clear to the mathematical agent (i.e. the person responsible for the idea, and not, in this case, the teacher), and 3) fixing mathematical work if it was required. In addition, the system of competence that was constructed in this classroom involved incorporating students' ideas about and experiences of mathematics, validating them by characterizing them as legitimate aspects of engaging in mathematical activity. Figure 2 outlines the ways elements of the classroom system came together in order to construct this particular version of competence.

4.2 Mathematical task

The task this class was working on is related to a key aspect of the Algebra Project Transition Curriculum, called a trip line. All students take an excursion together on public transportation (these students took the bus to a science museum), creating a common experience. From this experience, students progress from informal ways of representing and talking about their experience (drawing pictures, using familiar language), to more formalized ways of representing their experiences using mathematical symbols. One of the initial activities in this sequence involves representing a set of places that the students visited on the trip and to represent these as points on a line. With this representation, the class develops a concept of a *trip,* defined by four features: its starting point, its ending point, the distance (number of sites on the line to reach the ending point), and the direction. On this particular day, each student had already constructed representations of several trips (the assigned number was ten), and the task for each student was to collect a set of three trips from every student in the class. These were to be used subsequently to tabulate

<table>
<tr><td rowspan="6">Negotiation of</td><td rowspan="3">Participation structure</td><td>How an idea enters the common ground
Other people need to be able to understand</td><td rowspan="6">Leads to definitions of</td><td rowspan="2">What students are accountable for
Making sure that their symbols make sense</td></tr>
<tr><td>Who is expected to author or critique ideas
Other students</td></tr>
<tr><td>Norms of argumentation
students need to talk to and convince each other</td><td rowspan="3">Who students are accountable to
Other students and the teacher</td></tr>
<tr><td rowspan="3">Task as realized</td><td>Requirements for sense-making
doing mathematics—coordinating different records of the trip</td></tr>
<tr><td>Openness of the task
moderate—inventing symbols that others' could understand</td></tr>
<tr><td>Requirements for successful completion
Students need to be able to understand others' representations</td><td>What kinds of agency students can exercise.
Conceptual</td></tr>
</table>

Fig. 2 Model of the system of competence for a 6th grade classroom

frequencies of the different features, to estimate which starting point, ending point, distance, and direction were the most popular.

This activity included all the students, who were circulating through the room to obtain samples from each other. In one sense, the activity was quite open-ended. Each student's paper could (and most likely did) have a different set of trip records, depending on which of the other students' records they chose for their sample. At the same time, the features the students were to record were not open to individual student choice. Each record was supposed to represent the trip's direction, distance, starting point, and ending point. These features were represented differently by different students, and the activity required students to coordinate the meanings of different students' symbols for the four features.

In the terms of Stein et al. (2000), the cognitive demands of this task were to do mathematics—to justify their work rather than to retrieve something from memory or simply apply a procedure. In Pickering's (1995) terms, their activity required conceptual agency, as they were positioned as authors of symbols, and were accountable to each other and to the teacher for constructing symbols that would convey information about all four of the features of the trips that they represented.

4.3 Episode 1

This episode illustrates a moment of students being positioned with respect to agency and accountability. It is through multiple moments such as these that the larger system of competence gets constructed. Here students were being explicitly held accountable for making sure their symbols made sense, a process that they were positioned as being able to do using whatever means were most sensible—in other words, through exercising conceptual agency. Notably, sensibility was not left to individual students to arbitrarily decide, because students were also held accountable to their peers for whether or not their symbols indeed made sense. In the following episode this particular issue is explicitly negotiated. The episode began when a student, Callie, who was confused about the paper of another student, James, approached the teacher for help. Rather than helping Callie to understand James' work herself, the teacher involved James in the conversation so that he could explain the meaning of his symbols. What followed was a negotiation between the teacher and James to unpack what it means for a symbol to effectively communicate information.

1 T: (*standing with Callie, holding Callie's paper in her hand)* James, come here.
2 I'm just going to ask this question because I don't understand it. This symbol
3 three represents all this right? (*gesturing to his page*)
4 J: I was heading north.
5 T: How would I know that?
6 J: I don't know.
7 T: A symbol should communicate information to me, it should tell me
8 something. So this symbol tells me there were 3 stops?
9 J: No, no, no...
10 T: This symbol doesn't mean that?
11 J: Yes, that, that...
12 T: That means 3 right?
13 J: Right.
14 T: This means that that is where you finished. This means that is where you
15 started, so I need to know, this symbol means direction? I don't know
16 direction; how would I know which direction?
17 J: I'll change it. Darn you Callie.
18 T: She just wants you, why, ‘cause she just wants you to think a little bit? (*They all moved away. Later, T checked in with James again and he had made the symbol so that it could be understood*)

Notably, the system of competence that was constructed was quite open, with multiple acts being positioned as competent. The exchange that follows could have challenged the competence of either Callie or James. Callie could have been understood as lacking the needed competence to interpret a representation, or James could have been understood as lacking the needed competence to construct a representation with the required properties. Neither of these outcomes occurred. Instead, as the teacher and the two students talked together, both Callie's confusion about James' representation and James' identification of his mistake and revision of his representation were constructed as competent behavior—acceptable and expected parts of doing mathematics. Callie's role in the interaction was finally characterized as a constructive effort to influence James to “think a little.”

Accountability Accountability was explicitly negotiated in this interaction through moves of both the students and the teacher. Callie's initial move of asking the teacher for help served to include the teacher in the accountability system, as the person who could provide the resource that Callie needed in order to complete her work on the assignment. The teacher responded to this positioning by making accountability an explicit topic of conversation between the students. For example, in lines 1–2, the teacher asked James about his symbols. Her language, “I'm just going to ask this question *because I don't understand it*,” served to reinforce what students were supposed to be accountable for—making sure that their symbols made sense. What's more, the specific wording of the teacher's utterance functioned as a mitigated negative assessment. By framing her assessment as something she did not understand (rather than directly declaring its flaw) she positioned James with agency to interpret the representation he had constructed and a presumption of competence to do that. She positioned herself as someone who was interested in making sense of his work, rather than as someone who was simply evaluating his work. Her question also gave James the space to respond, rather than defend—by using a form that placed the problem with herself "*I* don't understand," she displaced her authority, and suggested that they were working on a goal (of sense-making) together. Thus although at this moment the teacher was the person who needed to understand, (and thus the person to whom James was accountable), together they constructed a broader meaning of what it meant to be accountable to someone else.

James responded by talking about the reality of his trip, that he was heading north. This made the question of what the symbol should represent relate back to the real trip the class took, an important step for making symbols make sense. In lines 5 and 7–8, the teacher problematized this for James, again reinforcing that the students needed to be accountable *for* making sure that their symbols made sense. But in addition, she was reinforcing that they needed to be accountable *to* others for communicating this information. She did this by taking what he said as given or correct, that the symbol showed a northern direction, and locating the problem in how his symbol communicated this information. These moves began to align what students were accountable for (making symbols make sense) with whom students were accountable to (each other, the greater world and the teacher). Final confirmation of the students' (or at least James') acknowledgement of this arrangement for accountability can be seen when James got angry at Callie (and not the teacher) for causing him to have to revise his work. The teacher further reinforced this assessment by noting that it was *Callie* who wanted him to "think a little bit."

Agency Regarding agency, this episode involved the cognitive demand of doing mathematics (rather than just applying procedures), as the teacher, Callie, and James were involved in a negotiation about the meaning of a symbol that James had authored. The students were vested with conceptual agency in their authoring of the symbols and with the concurrent responsibility of revising those symbols if they weren't adequate. Notably, the teacher's expectation that James revise his symbols, and the form of her assistance (helping him to see where he was not communicating clearly without giving any indication of how he should deal with that problem), kept the cognitive demand of the task quite high and preserved the possibility that students could complete the task by exercising conceptual agency.

The teacher's framing of the activity, with her emphasis on the requirement of mathematical thinking, fits Pickering's idea of conceptual agency. For example, in line 17, James accepted the work that he had to do in order to make his symbols make sense, but then complained about Callie, the student who had initiated the conversation. The teacher's response again positioned the task and James's role in the task as requiring conceptual agency, by framing what it meant for another student to question your work with "she just wants you to think a little bit." This positioned the mathematical task itself as something that takes some work, that is not immediately easy to do, but indeed as something doable. Later in the class, James did follow up, and made his symbol make sense. In the course of this interaction, both Callie and James were positioned as being competent at creating and understanding meaningful symbols. In addition, the norms that symbols should make sense, and that it is acceptable to ask other students to make their symbols make sense, were reinforced.

4.4 Episode 2

In this classroom, what counted as competent mathematical behavior started from students' experiences of doing mathematics, which were positioned as a legitimate aspect of mathematical work. What it meant to do mathematics in this classroom arose from negotiations between the students and the teacher, rather than a teacher-driven model of mathematical behavior which students were left either to accept or reject. In this episode, the teacher worked with one student to construct his *experience* of doing mathematics as a legitimate aspect of mathematics—being confused and finding things to be difficult. The episode begins when the teacher approached a student, Michael, who was sitting quietly at his table, not noticeably working.

1 T: You all right?
2 M: Aren't. It's stressful.
3 T: What makes it so stressful? Why, why are you so stressed?
4 M: Because you got to draw all that.
5 T: Oh, and that's stressful?
6 M: Why do you have to put symbols that you have to draw. Why can't they just write stuff?
7 T: Would it be less stressful if you just wrote it?
8 M: Yes
9 T: But wouldn't you still have to write down what they wrote?
10 M: Yeah.
11 T: So, would that really decrease the stress?
12 M: No.
13 T: So, what would be a good way to really decrease the stress?
14 M: Just by doing it.
15 T: Doing it. We might think of some other way of doing it because even Riana
16 was saying that it's just too much.

As the teacher approached Michael, he appeared to be off-task. Her opening question suggested that he was doing something different than she expected him to be doing, but did not deliberately position him as doing something wrong. Saying “are you all right” signals that there is reason to believe that something is wrong (the student isn't doing what was asked), but does not assert that the rationale for this behavior is willful disobedience. This is consistent with the way that the teacher approached James in the previous episode, wherein she positioned him to explain what he was doing, rather than defend it. Once again, these teacher moves serve to contribute to a very open system of competence, where a range of behaviors are considered to be part of the activity of doing mathematics successfully.

Accountability With respect to a system of accountability, this interaction reinforced the expectation that students are not simply accountable to the teacher for engaging with mathematics, nor are they accountable *for* engaging with mathematics in a way that the teacher outlined. In asking Michael to explain how he was feeling, the teacher created a space for Michael to re-negotiate what specifically he was accountable for, and what doing this assignment should look like. In essence, she created an opportunity for him to explain how his behavior was acceptable and competent. Michael took up this opportunity and responded by making his feelings about doing the mathematics an issue they both acknowledged — that he was not OK, because the work was stressful. In line 3, the teacher started pinpointing what was stressful in order to begin to solve the problem. This move both served to accept the student's response as a legitimate, valued utterance, and demonstrated that being stressed was an acceptable part of doing mathematics. In lines 4–7, we see the problem being defined as one of symbolizing. The student said that if other students had just written what they meant instead of using symbols for it, his task would have been easier. This is an interesting problem because of the way it brings together mathematical and emotional issues. The student was feeling stressed because of the way the mathematical task was structured; it asked him to figure out meaning rather than just write something down. “Why can't they just write stuff?” seems like a very typical and important question that reform teachers may need to answer. Akin to “Why can't you just tell us the answer,” this question is directly about what the students' role as a mathematical learner was and what it took to be competent in this classroom. The student presented himself as wanting to be competent, but finding it difficult. Although the teacher guided Michael to say that he needs to actually do the assignment in order to find it to be less stressful (lines 13–14), she continued to legitimate his claim of finding the assignment to be too much when she noted, in lines 15–16, that they needed to think of a different way to complete the assignment.

Agency In this episode the discussion between the teacher and student painted a picture of agency as going beyond simply practicing procedures (disciplinary agency). Instead, having to replicate (and understand) other people's work required exercising conceptual agency as well, though the interpretation of the meaning behind different symbolic representations. In this way, students' had opportunities to engage in the activity of meaning-making, and were expected to do so despite the fact that the activity was challenging.

Competence, in this episode, was constructed through the teacher's effort to (re)engage Michael in the mathematical activity of interpreting and recording the symbols of other students' records. Her action of encouraging him to become engaged presupposed his competence to work productively on the task. The conceptual agency in which he was thereby constructed as being competent to engage involved participation in the class's joint effort to develop a collection of symbols that would successfully convey the information they needed to determine the frequency of their choices of features of trips.

4.5 Replicating and practicing: An episode in an eighth grade classroom

This next episode comes from another classroom, an eighth grade algebra class. We propose that here, too, the system of competence was the result of alignment between agency and accountability. However, in this case the classroom was organized very differently, with both accountability and agency being defined in ways that were distinct from observations of the sixth grade classroom.

In the eighth grade classroom, participating competently meant following directions and completing problems using the procedures or methods that had been modeled. The process was straightforward and could be completed quickly and efficiently. This classroom was structured so that students had ample opportunities to participate competently—the teacher showed them one way to solve a problem, and then provided occasions to practice that method on other problems. She allowed them to work in groups so that they could help each other to complete the problem, and assisted them by supporting them in completing the problem in the way that she had illustrated earlier. This contrasts with what was constructed as competent mathematical behavior in the episodes we discussed from the 6th grade classroom, where students were required to ensure that their ideas made sense to their peers.

The excerpt illustrates a moment in which the system of competence as described above was constructed and reinforced. In this excerpt, the focus of the mathematical activity—what students were accountable for—was learning to use the correct procedure at the right time. Thus, they were primarily afforded opportunities to exercise disciplinary agency (Pickering 1995) through the application of a method of calculating proportionality, with some attention to its meaning in the situation involving similar triangles on a Cartesian graph. The source for determining whether the students were correct—who students were accountable to—was the teacher. These positionings were reinforced by both the teacher and the students, and can be seen in the kinds of questions they asked each other, the expectations they held of each other, and the times when confusion arose. Figure 3 outlines the ways elements of the classroom system came together in order to construct this particular version of competence.

4.6 Mathematical task

The task that the teacher assigned involved finding an unknown point on a line without knowing the equation of the line. She introduced a method that involved using similar triangles. The teacher's presentation was couched in terms of the mechanics of a procedure, and she checked in with her students to make sure they were following what she was doing and where the information she was using came from. In this interaction, the teacher asked questions that created a space for students to think about how the information that they had fit into this procedure. They could demonstrate that they were following the explanation, and they could demonstrate that they were making sense of the connections between a graph and similar triangles. At the end of her presentation, the teacher gave instructions for the rest of the period:

> So the idea class is to use ratio and proportion, ratio of similar triangles to solve for values of ‘y.’ This is related to our lesson today, in eighty-one through eighty-four. You have to look for…in eighty-one through eighty-four, eighty-one and eighty-two are similar

<table>
<tr><td rowspan="6">Negotiation of:</td><td rowspan="3">Participation structure</td><td>How an idea enters the common ground
Through careful explanation by an authority</td><td rowspan="6">Leads to definitions of</td><td rowspan="2">What students are accountable for
Using procedures accurately</td></tr>
<tr><td>Who is expected to author or critique ideas
The teacher</td></tr>
<tr><td>Norms of argumentation
Students need to convince the teacher that they understand</td><td rowspan="3">Who students are accountable to
The teacher</td></tr>
<tr><td rowspan="3">Task as realized</td><td>Requirements for sense-making
Procedures without connections—practicing</td></tr>
<tr><td>Openness of the task
closed—procedures were specified</td></tr>
<tr><td>Requirements for successful completion
students need to have correct answer (as designated by teacher)</td><td>What kinds of agency students can exercise.
Disciplinary</td></tr>
</table>

Fig. 3 Model of the system of competence for a 8th grade classroom

problems where in you are going to graph. Eighty-three and eighty-four are problems that use ratio and proportion...those are simple problems you don't need to graph it.

The activity preceding the instructions, and the instructions themselves, set up a well-defined space for student participation: both what the students would be accountable for (solving the problems by using the method that was just demonstrated), and who they would be accountable to for demonstrating this knowledge (the teacher) was arranged. In this way, the task could be classified as *procedures without connections*; students were told to review the steps of a procedure and practice them on new problems. They were not required to interrogate the *meaning* of these procedures, however, or why they were useful in solving the problems posed. This was reinforced both by the completion of an "introductory" problem by the whole class, and by the explicit instructions that the teacher gave about how to do the problems that were going to be assigned. The teacher had introduced some guidelines for what it would look like to solve these problems competently—use the procedures that she had just demonstrated.

4.7 Episode 3

In the following episode the students were working on the problems that had been assigned to their group. The teacher came to check in with this group as they worked on the problems. In this interaction, whom the students were accountable to for "settling" their mathematical questions or issues, and what they were accountable for was well-aligned between the students and the teacher. The students looked to the teacher for confirmation and help about the mechanics of the procedure, and the teacher provided the kind of help that they needed to both reinforce their progress with the procedure, and guide their mathematical work vis-à-vis those procedures. As a consequence, the competence that the students were able to display was limited to providing answers to relatively low-level questions from the teacher, involved in the arrangements of symbols in correct algebraic form.

1 T: Yes Darcy, you have a question?
2 D: Is this right? It is like she is asking me
3 T: You want to discuss it with your team first?
4 D: No, first we want you to discuss it with us (*whining tone*)
5 T: Oh no, you have to work as a—wait a minute (*students protest*) OK what is
6 the question about?
7 D: It's just saying write an equation. But=
8 T: =mmm hmm. So how do you write it. This is x, this is x how do you write
9 an equation for number of gallons used.
10 D: X divide by 20 equals G
11 T: X equals...what is x divided by?
12 D: X over 20 is equal to G
13 T: OK that's good. That's an equation right?

In this excerpt, alignment between agency and accountability, and their role in defining a system of competence can be seen as both shaping this interaction and being reinforced through the interaction.

Accountability In the exchange in the first five lines, work was getting done to establish who students were accountable to in terms of establishing that mathematical progress has been made. In line 3, the teacher's suggestion that they discuss with their group created a space for students to become accountable to each other at some level. However, at this point the teacher did not use very authoritative language (as she used later in the interaction). For example, she did not say: “ask your teammates,” or “how could you know for yourself?” Rather, she suggested, in the form of a question, that the student work with her team first. This could have been constructed as an implied request, but the students treated it as an offer, which they declined. The students insisted that they would rather work with her, and it took relatively little effort on the part of the students to convince the teacher to help them (one exchange). This exchange reinforced the teacher's role as the authority, accomplished by the students themselves, as they positioned her as the person whom they would most like to work with. With this resistance, the teacher immediately returned to a structure in which she was guiding the students though the steps of the problem, and letting them know when they were correct. In the next lines the teacher and students negotiated and reinforced what the students needed to be accountable for doing or knowing. Starting in lines 8–9, the teacher began asking questions with known answers that specifically targeted the *doing* of procedures, such as: “How do you write it?” This question structured the task as one that was about disciplinary agency, with students being positioned as being competent at writing equations. In addition, the person they were accountable to (the person they needed to persuade that they indeed knew how to execute the procedure), was the teacher. This can be seen in the teacher's response to Darcy's answer, in line 11—asking another question about the mechanics of the equation, “testing” her understanding. When Darcy answered again, the teacher was apparently satisfied (line 13), therefore reinforcing the expectation that the students needed to make sure that they convinced her before moving on. These exchanges served to locate the authority for settling mathematical issues with the teacher, rather than with individual students or their classmates.

Agency There is evidence of a system of competence that was guiding the interaction, as seen in Darcy's question “is this right?” and in the way the teacher took up this question. “Is this right?” communicated that the activity, at least at some level, was about finding the correct answer. This particular understanding of competence, focused on disciplinary agency, was further reinforced in the interaction that ensued between the students and the teacher—with the students asking for help, and the teacher providing *a specific kind* of help that seemed to be useful to the students. The way that accountability was defined in this interaction was well-aligned with the kind of agency that students had an opportunity to exercise. As previously stated, the task, as introduced and as enacted by the students and the teacher, created opportunities for students to practice using a particular method with new numbers. Students were not asked to make sense of the method, or to experiment with other methods. Thus, there were limited opportunities for students to exercise conceptual agency. However, this worked well in a system wherein students were only held accountable for having used methods accurately. What students were expected to do, and supported in doing, was both clearly and consistently reinforced.

This interaction, which seemed to be organized in terms of “getting the right answer,” ended up constructing a more specific version of competence as defined by a correct execution of a procedure. For example, when in line 13 the teacher said: “OK, that's good!” she reinforced the student's competence for executing a procedure correctly. Competence continued to be defined in this way, as Darcy and the teacher continued to work through the problem, locating and correcting Darcy's misunderstanding. The teacher took an even more active role in this interaction, helping Darcy to work though the equation that she had just constructed, and helping to prove to her why the equation was sensible.

The episodes from the sixth and eighth grade classrooms are useful to consider alongside each other. They are both examples of classrooms in which the task, students, and teacher are well aligned in their expectations of each other, and their fulfillment of those obligations. However, in these episodes, we can see that the mathematical behaviors that actually were constructed as being "competent" looked quite different.

5 Discussion

We have presented a framework for considering competence of students as a characteristic that is constructed in classroom interaction, and shown illustrative analyses that support some conjectures about the nature of competence when it is viewed this way. These micro-analyses of moment-to-moment interaction in two middle school mathematics classrooms have enabled us to better understand the characteristics of classroom interaction, and how episodes of activity are distributed between the task, teacher, and students. First, opportunities for students to be understood as being competent depend on the tasks that they are assigned to work on, and on the agency and accountability with which they are positioned to do that work. Echoing one of the findings of Pope (2001), it seems that students must learn that "doing school" looks different from one classroom to the next, from one year to the next. Specifically, the behaviors that are constructed as competent in one classroom can be dramatically different from those in another classroom—even classrooms that ostensibly cover the same subject matter[5]. Thus, when determining whether or not students are "proficient," it is essential to characterize not just individual students' accomplishments, but their accomplishments in the context of the opportunities they have had to develop that proficiency (Greeno and Gresalfi 2008).

Regarding agency, the kind of competency that can be constructed is related to the cognitive demands of the task (Stein et al. 2000), and the kind of agency (conceptual or disciplinary, Pickering 1995) that the task affords. In these analyses, we have seen that the structure of the task itself affords different opportunities for students to engage with content in different ways. While the task structure of course does not *determine* how students interact with the content, it provides a framework that shapes the kinds of agency that are most accessible to students as they move productively through the task or resist being engaged in the task. This had implications for the mathematical opportunities that were created for the students—particularly in terms of mathematical sense making. In the 8th grade classroom, students were participating competently as long as they were using the right method. The teacher had explained how to use the method, and helped the students use it in assigned problems, but they did not engage in discussion about how use of the method made sense. In contrast, in the 6th grade classroom episodes students' act of sense-making got constructed as competent, although how they made sense of ideas at times took different forms (working hard, asking other students, etc.; Fig. 2).

The way that accountability is distributed, including what students are accountable for and whom they are accountable to, contributes to their opportunities to be understood as

[5] It is important to note here that although we have focused on how competence is constructed at the level of the classroom, this does not mean that all student behaviors in the classroom are likely to be positioned in exactly the same way. Although this was not the focus of our analysis, it is likely that individual experiences, such as students' histories in the classroom, their sensitivities to particular kinds of opportunities to participate, or even membership in particular racial, ethnic, or gender groups, could shape the likelihood of *particular* students being reified as competent (or not; c.f. Jungwirth 1991).

competent in working on the task and evaluating their progress. The importance of accountability is recognized widely in current educational discourse, and we submit that unpacking what accountability means in interaction is a crucial dimension of understanding what can, and will, be learned. Cooperative students work to meet the goals set for them—what they are accountable for, when clearly laid out, will impact what they ultimately do. Likewise, to whom students are accountable in demonstrating achievement arguably shapes their ideas about what it means to know something, and who has the right to determine when something is known. Unpacking the word "accountability" to attend to the specifics of what, and to whom, students are accountable, seems to be an important next step in this and other analyses of mathematics learning.

5.1 Implications

What gets constructed as "competent mathematical behavior," in a classroom is inseparable from what students actually have opportunities to learn about mathematics. This is supported by many researchers who have sought to re-cast the relationship between the individual and the system in which he or she is participating (Boaler 1999; Brown and Campione 1994; Cobb, Wood and Yackel 1992; Greeno 1991; Greeno and MMAP 1998; Lave 1988, 1991). Specifically, researchers have pushed to consider learning as a process through which individuals participate in the practices of a particular activity by interacting with the resources (other members, artifacts, etc.) in the setting. Rather than focusing on some amount of knowledge that an individual can acquire, this definition of learning shifts the focus to the kinds of practices in which people have an opportunity to participate, including the ways people come to relate to each other within a particular activity setting. As we have demonstrated in the preceding analyses, the ways that competence gets constructed in classrooms directly impacts the kinds of opportunities to learn that are presented to students, and thus both the content and practices with which they can engage. This suggests that efforts to understand *whether* students are learning should not be attempted without a critical focus on *what* students might be able to learn, given the organization of the classroom settings in which they are participating.

Finally, these analyses have implications for an ongoing concern in our research, how to create opportunities for *all* students to be successful. The contrast between the cases that were shared point to two promising avenues that we plan to consider in future research. First, we shared episodes in classrooms in which the expectations of the teacher were made clear, and in which those expectations were supported by multiple elements of the classroom system. Classrooms which provide clear expectations, wherein students know what is expected of them and are supported in meeting those expectations, has been frequently cited as necessary to equitable teaching (Delpit 1995; Ladson-Billings 1997). Continued attention to the role of alignment of expectations which guide but do not limit[6] mathematical activity deserves continued attention, as we struggle to understand how to give all students access to the expectations of tasks without limiting their ability to engage creatively with such tasks.

The second issue that has emerged from this analysis is the explicitness of the negotiation that takes place between the teacher and the students about what it means to be

[6] Of particular concern is an issue raised by Stein et al. (2000), and others, who note that it is far too common for teachers, in their attempt to make their expectations clear to their students, to limit the cognitive demand of tasks. In such cases, students end up following a set of prescribed rules or steps, rather than engaging in a task that involves meaning-making.

competent. Although in any classroom, what counts as competent is clearly constructed in interaction between the teacher and the students, the extent to which space is made for students' experiences of mathematics to influence the mathematical agenda was dramatically different in the episodes we shared. Specifically, the system of competence in the eighth grade classroom was not especially responsive to students' own ways of participating, but rather reinforced a process of engaging with mathematics in one particular way. When a student was confused, this was taken as a problem, and solved by helping the student to become aligned with an official way of doing mathematics. In contrast, the system of competence seen in the sixth grade classroom involved ensuring that students' experiences of doing mathematics were incorporated into the official rhetoric of what it means to do mathematics (cf., Hand 2003). These differences suggest that further consideration of the role of students' original contributions to the system of competence that gets constructed in a classroom is another promising avenue for further research.

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