AFRICAN DRUMS AND RATIOS CURRICULUM

PULSE & MEETING NUMBER MODULE

STUDENT WORKBOOK
3RD EDITION

Benjamin Moynihan
John Belcher
The Algebra Project, Inc.

AFRICAN DRUMS & RATIOS CURRICULUM-PULSE & MEETING NUMBER MODULE STUDENT WORKBOOK, 3RD EDITION

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AD&RC PULSE & MEETING NUMBER MODULE

DESCRIPTION

The African Drums & Ratios Curriculum (AD&RC) was developed, beginning in 1991, at the Algebra Project. AD&RC modules use activities drawn from African and African Diasporic (African-American, Afro-Caribbean, Afro-Brazilian, etc.) drum traditions as a point of departure for experiential learning of mathematics. The AD&RC uses these concrete events to help students construct understanding of key mathematics concepts that provide a foundation for proportional reasoning. The pedagogical framework for AD&RC modules is the Algebra Project's Five-Step Curricular process, which Robert P. Moses developed for the Algebra Project's "Transition Curriculum." The "Pulse & Meeting Number" module is one of two African Drums & Ratios Curriculum modules, which also includes "Drum Making."

AD&RC lessons emphasize a variety of observation, reflection, communication, and standard and non-standard symbolic representation strategies, and can be aligned with national mathematics and national arts-in-education standards. The modules are intended to be used as curricular interventions, integrated with other mathematics and interdisciplinary materials. The teaching of AD&RC modules is greatly enhanced when a drummer, proficient in one or more African Diasporic drumming traditions, is involved. AD&RC materials are appropriate for late elementary classroom settings, as well as in after school and/or summer programs, where, suitably adapted, a wider range of students may be served.

DEVELOPERS AND COLLABORATORS:

Benjamin Moynihan, M.Ed., has coordinated the AD&RC program since its inception, and John Belcher, MA, M.Ed., was co-coordinator from 1993-2000. Over the years, many people have helped to develop, refine and implement AD&RC materials; they are listed on the following page.

Algebra Project Founder & President Robert P Moses provided the impetus for the AD&RC program. Algebra Project Trainers William Crombie and Staffas Broussard have provided invaluable guidance, support and co-development assistance through these years.

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TO STUDENTS AND TEACHERS

The African Drums & Ratios Curriculum (AD&RC) is an experientially-based approach to learning mathematics. The AD&RC can be taught in classrooms, after school programs, or summer programs by a teacher and a drummer—ideally with a drummer that is proficient in one or more African Diasporic drumming traditions. The "Pulse & Meeting Number Module" is one of two AD&RC modules, which also includes "Drum Making."

What you observe and discover in the African Drums & Ratios Curriculum is a very important part of this adventure. The lessons depend heavily upon your ideas and your ways of thinking. Sometimes you will be working on your own, sometimes in teams, and sometimes as a whole class. In each of these situations, your ideas and input are greatly valued and, indeed, are necessary for a successful class.

If you have any questions, comments, or suggestions, please contact us at:

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ABOUT THE AFRICAN DRUM & RATIOS CURRICULUM

We welcome you as a participant in the African Drums and Ratios Curriculum! For our journey into African Diasporic drumming and mathematics you will be asked to think and to do things. In fact, your thinking and doing are the most important parts of this curriculum. These will take many forms. Some of the ways in which you will be asked to think and to do things will be by:

- Collecting Information
- Listening (Using Your Ear Drums!)
- Making Clear Descriptions
- Considering Different Points of View
- Comparing and Contrasting Ideas
- Identifying Different Ways of Solving Problems, and,
- Representing Ideas in Different Ways.

Some of these forms are already familiar to you. Other kinds of thinking and communicating may be new to you and to your classmates.

Each of these different ways of thinking is a kind of skill.
Any skill requires practice in order to become good at doing it.
You will get many opportunities to practice these skills in
the African Drums & Ratios Curriculum.

PULSE & MEETING NUMBER MODULE

TABLE OF CONTENTS

SUPPLEMENTAL LESSONS FOR THE PULSE & MEETING NUMBER MODULE

LESSON 1
PULSE

LESSON OVERVIEW: In this lesson, we will explore **pulse**, an important concept that occurs in music, mathematics and science. We will start by working with our own heartbeats and listening for pulses in music.

FINDING A PULSE

Class Discussion: Have you ever gone to the doctor's office or the hospital and had your pulse checked?

Do you remember how the doctor or nurse took your pulse?

Have you ever checked your pulse in other situations, such as in physical education class?

The class will discuss and demonstrate different ways of finding your pulse. Use the space below to draw pictures or write down ideas from the discussion.

Illustrations

Checking our pulse can tell us information about
_____ .

PLAYING YOUR PULSE ON A DRUM

Individual Activity: Check for your pulse in one of the ways that the class discussed. Memorize the feel of your pulse first and then play it on a drum.

Individual Work: Describe your pulse in three different ways.

- Using pictures - draw a picture of your pulse.
- Using writing - write a description of your pulse.
- Using sound - play your pulse on the drum

Draw a Picture of Your Pulse:

Student: _____ Team: _____

Write a Description of Your Pulse:

_____

_____

_____

Drum Sound Description of Your Pulse: Using your memory, play your pulse on your drum.

Team Work:

- Each team member plays his or her pulse for the whole team.
- While each member plays her or his pulse, the rest of the team copies.
- Use chart paper to make lists, and a team diagram, showing some of the ways that your team members' pulses are alike and different.

Class Work: Teams share their work with the rest of the class.

- Team members first check their pulses and then all play them at the same time.
- The class makes lists of the ways that class members' pulses are alike and different.

As teams report out, use the table below to write down ways that class members' pulses are alike and different.

Ways That Pulses Are Alike	Ways That Pulses Are Different

FEATURES OF A PULSE

When you made your descriptions of your pulse and found ways that your classmates' pulses were alike and different, you were thinking about some of the different features of a pulse. In this lesson, we introduced two features of a pulse that we will use in the lessons that follow:

(1) the speed, 'tempo', or rate of a pulse (how fast the pulse beats occur in a pulse);

(2) the even spacing of pulse beats in a pulse.

Optional Class Activity: The class may want to invite someone from a medical profession (a medical student, a nurse, or a doctor) to talk with them about heartbeats and other body rhythms. Students could listen to their heartbeats with a stethoscope and use the drums to play what they hear.

Class Discussion: Give other examples of pulse. Think of how pulse can be found in people, places and things. The class will gather information about, and then share, different examples of **pulse**. For example, sometimes we speak of *the pulse of a city*. Use the space below to record ideas or draw pictures from the discussion.

Pulse in Music

Class Demonstration: The drummer and teacher will demonstrate examples of pulse in percussion music.

Individual Work: In the space below, draw a picture of, and write a description about, the percussion music demonstration.

Written Description of the Percussion Demonstration:

Class Work: The class will listen to various examples of recorded and/or 'live' music and identify a pulse that goes with each selection. You may express a pulse that goes along with a selection by clapping pulse beats, or some other way that you invent.

Lesson 1 Vocabulary

Write down the new word that you learned in this lesson. Use each of the words in a sentence.

LESSON 2
PLAYING GROUPS OF PULSE BEATS

Class Work: The drummer or teacher will ask for a volunteer to play their pulse on a drum. While the pulse is being played, the drummer/teacher will ask you to begin grouping *pulse beats* in different ways:

- Clap once for every two pulse beats you hear in the pulse. Clap only on "one" while you count:
  one, two, **one**, two, **one**, two, etc.
  along with the pulse that is being played.

  This is an example of a **pulse beat group**
  with ***two*** **pulse beats per group**.

Note to the class: You may use a Cowbell, Clavé, or other percussion instrument to play on the one pulse beat.

- How would you play a **pulse beat group** with ***three*** **pulse beats per group**? Clap once for every three pulse beats you hear. Clap only on **one** while you count: **one**, two, three, **one**, two, three, **one**, two, three, etc. along with the pulse that is being played.

- Now try a **four-pulse beat group**. Clap once for every four pulse beats you hear. Clap only on **one** while you count: o**ne**, two, three, four, **one**, two, three, four, **one**, two, three, four, etc. along with the pulse that is being played.

- Now try a ____ - pulse beat group. (Choose another pulse beat group different from those that we have played thus far.) Remember to clap just on the "**one"** of your pulse beat group along with the pulse that is being played.

Team Work: Teams practice playing pulse beat groups of various sizes. Take turns being the pulse player--this person will play an even pulse on a drum. The other teammates choose a pulse beat group size (5, 6, 7, 8, … - pulse beat group) and practice clapping on the "one" of that pulse beat group along with the pulse that is being played. Please choose pulse beat group sizes that the class has NOT tried already.

Our team chose the following pulse groups for practice:

- A __ Pulse Beat Group.
  This group has ___ pulse beats per group.
- A __ Pulse Beat Group
  This group has ___ pulse beats per group.
- A __ Pulse Beat Group
- This group has ___ pulse beats per group.
- A __ Pulse Beat Group.
- This group has ___ pulse beats per group.

Individual Work: Use the space below to draw a picture and write a description of one of the pulse beat groups you played with your teammates.

___-Pulse Beat Group

(Fill in the size of the pulse beat group that you chose)

Write a Description of Your Pulse Beat Group:

Class Discussion: Give other examples of pulse beat groups. Think of how pulse beat groups can be found in people, places and things. The class will gather information about, and then share, different examples of **pulse beat groups**. For example, one week could be thought of as a kind of pulse beat group.

Pulse Beat Groups in Percussion Music

Class Demonstration: The drummer and teacher will demonstrate examples of pulse beat groups in percussion music

Individual Work: In the space below, draw a picture of, and write a description about, the percussion music demonstration.

Written Description of the Percussion Demonstration:

Class Work: The class will listen to various examples of recorded and/or 'live' music and identify pulse beat groups that go with each selection. You may express a pulse beat group that goes along with a selection by clapping or using some other way that you choose.

Lesson 2 Vocabulary

Write down the new words that you learned in this lesson. Use each of the new words in a sentence.

LESSON 3
EXPERIMENTING WITH PULSE BEAT GROUPS

LESSON OVERVIEW: We will begin a series of experiments to see what happens when you play different pulse beat groups at the same time. In this lesson, we will experiment with playing a 3-pulse beat group and a 4-pulse beat group at the same time.

Individual Work: Below, make a **conjecture,** about will happen when you play a 3-pulse beat group and a 4-pulse beat group that begin together and use the same pulse.

Note to Students: When mathematicians do their work with numbers, patterns, and shapes, they often make an educated guess, or a prediction, about what they expect to find. Mathematicians call this type of thinking, "**making a conjecture.**"

Conjecture --
After beginning together on the first pulse beat, the 3-PBG and the 4-PBG _____ meet again.
(will/will not)

My reason(s) for making my conjecture are:
(You may also include drawings and/or diagrams to back up your conjecture.)

Class Discussion: Discuss your conjectures and reasons from the previous page. Record three reasons from this discussion.

1.

2.

3.

Experimenting with Pulse Beat Groups

Class Experiment: The class will try an experiment to explore the questions on the previous page. This experiment requires the following:

(1) a person to play a pulse;

(2) a person to play a 3-pulse beat group;

(3) a person to count the 3-pulse beat group out loud to verify accuracy

(4) a person to play a 4- pulse beat group, and;

(5) a person to count the 4-pulse beat group out loud to verify accuracy.

Teacher/Drummer: Give a signal so that everyone can begin together.

Individual Work:

Experiment Result

After beginning together on the first pulse beat, the 3-PBG and the 4-PBG _____ meet again.
(did/did not)

Were you surprised by this result? _____

Why or why not?

Class Discussion: Discuss the results of the experiment and any ways that you and your classmates were surprised by what happened. Take notes in the space below.

Pulse and Pulse Beat Groups in a Drum Song

Class Demonstration: The drummer and teacher will demonstrate an example of percussion music that includes a pulse and different pulse beat groups played at the same time.
Individual Work: In the space below, draw a picture of, and write a description about, the percussion music demonstration.

Written Description of the Percussion Demonstration:

Class Work: The class will listen for combined pulse beat groups in various examples of recorded and/or 'live' music.

Lesson 3 Vocabulary

Write down the new words that you learned in this lesson. Use each of the new words in a sentence.

3-PBG and 4-PBG Meeting Number Chart

Pulse Count	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
3-PBG	1 X	2	3	1 X	2	3	1 X	2	3	1 X	2	3	1 X	2	3	1 X	2	3	1 X	2	3	1 X	2	3	1 X
4-PBG	1 X	2	3	4	1 X	2	3	4	1 X	2	3	4	1 X	2	3	4	1 X	2	3	4	1 X	2	3	4	1 X
MN-PBG	X																								

Pulse Count	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
3-PBG	2	3	1 X	2	3	1 X	2	3	1 X	2	3	1 X	2	3	1 X	2	3	1 X	2	3	1 X	2	3	1 X	2
4-PBG	2	3	4	1 X	2	3	4	1 X	2	3	4	1 X	2	3	4	1 X	2	3	4	1 X	2	3	4	1 X	2
MN-PBG																									

Pulse Count	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
3-PBG	3	1 X	2	3	1 X	2	3	1 X	2	3	1 X	2	3	1 X	2	3	1 X	2	3	1 X	2	3	1 X	2	3
4-PBG	3	4	1 X	2	3	4	1 X	2	3	4	1 X	2	3	4	1 X	2	3	4	1 X	2	3	4	1 X	2	3
MN-PBG																									

Individual Work:

What Meeting Numbers did you find?

Start # -- ____ , 1st MN -- ____ , 2nd MN -- ____ , 3rd MN -- ____ , 4th MN -- ____ , 5th MN -- ____

How do the meeting numbers that you found using the chart compare with those that the class found by doing the meeting number experiment? (Were they the same or different?)

What new pulse beat group do the 3-PBG and a 4-PBG create, based on the beats when they play together? _____

In other words, if we count "one" beginning with the Start Number and continue to count higher and higher until we reach the 1st Meeting Number (13) when we count "one" again and continue to count higher and higher until we reach the 2nd Meeting Number (25) and so on, what size pulse beat group would we be counting?

What strategy did you use to come up with this answer?

Class Work:

Meeting Numbers (MN's) for a 3-Pulse Beat Group and a 4-Pulse Beat Group:

Start #	1
1st MN	13
2nd MN	25
3rd MN	37
4th MN	49
5th MN	61

One strategy for figuring out the Meeting Number Pulse Beat Group (MN-PBG) is to use the chart on page 23 to count the pulse beat group formed by the Xs in the bottom row.

Note to teacher(s) and students: The class as a whole should count out the pulse beat group formed in the bottom row. Begin by counting "one" on the Start Number. Count higher and higher until you come to the next Meeting Number when you count "one" again and so on.

Meeting Numbers (MN's) for a 3-Pulse Beat Group and a 4-Pulse Beat Group:

Start #	1
1st MN	13
2nd MN	25
3rd MN	37
4th MN	49
5th MN	61

Another strategy for figuring out the Meeting Number-Pulse Beat Group is to "take the difference of," or subtract, consecutive Meeting Numbers. For example,

The 1st Meeting Number (13) minus the Start Number (1) is equal to ___ . In other words, 13 – 1 = ____.

The 2nd Meeting Number (25) minus the 1st Meeting Number (13) is equal to ___ . In other words, 25 – 13 = ____.

The 3rd Meeting Number (37) minus the 2nd Meeting Number (25) is equal to ___ . In other words, 37 – 25 = ____.

What is your prediction for the difference between the 27th Meeting Number and the 26th Meeting Number? ____
(In other words, the 27th MN – the 26th MN = ____.)

Team Work:

Make Unifix cube models of a 12-Pulse Beat Group, a 3-Pulse Beat Group, and a 4-Pulse Beat Group. Make enough copies of the 3-Pulse Beat Group model to fit evenly into the 12-Pulse Beat Group model. Also, make enough copies of the 4-Pulse Beat Group model to fit evenly into the 12-Pulse Beat Group model.

Draw a picture of your Unifix cube models 12-Pulse Beat Group, a 3-Pulse Beat Group, and a 4-Pulse Beat Group. Your picture should show how many 3-Pulse Beat Groups and how many 4-Pulse Beat Groups fit into a 12-Pulse Beat Group.

12-Pulse Beat Group, 3-Pulse Beat Group, and 4-Pulse Beat Group

More Questions About Playing a 3-Pulse Beat Group and a 4-Pulse Beat Group Together with a Pulse

Individual Work: Think about the 12-Pulse Beat Group again and answer the questions below. You may also use the Unifix models and/or pictures that you created for the work on the previous pages.

Let us use the following notation:
a pulse beat = PB;
a pulse beat group = PBG

How many times does a 3-PBG (3-Pulse Beat Group) repeat before the second meeting number? ____

How many times does a 4-PBG (4-Pulse Beat Group) repeat before the second meeting number? ____

Team Work: Complete the table below for the number of repetitions (repeats) of a 3-Pulse Beat Group and a 4-Pulse Beat Group for the first five meeting numbers.

In the far right column in the table, fill in the number of repetitions of the pulse beat group that is formed by the meeting numbers of the 3-Pulse Beat Group and the 4-Pulse Beat Group-- which is a 12-Pulse Beat Group.

REPETITION NUMBERS TABLE
(3-PULSE BEAT GROUP, 4-PULSE BEAT GROUP, 12 -PULSE BEAT GROUP)

Meeting #'s (MN's) for 3&4-PBGs	3-pulse beat group repetitions	4-pulse beat group repetitions	12-pulse beat group repetitions
1st MN: ___			
2nd MN: ___			
3rd MN: ___			
4th MN: ___			
5th MN: ___			

Team Discussion: What patterns do you find in the information in the table on the previous page? Please use the space below to write your answers:

Class Discussion: Teams report out the data from their Repetition Number Table on the previous page and their answers about the patterns they found in the information in the table. Use the space below to take notes on the class discussion:

Team Work: Complete the problems in the Repetition Numbers Tables below. The teacher will give you extra paper to do any calculations, draw any pictures or make any charts. Show all of your work. Each person on your team will be responsible for explaining your answers and how you came up with them.

PROBLEMS FOR A REPETITION NUMBERS TABLE (3-PBG, 4-PBG, 12 -PBG)

Meeting #'s (MN's) for 3&4-PBGs	3-pulse beat group repetitions	4-pulse beat group repetitions	12-pulse beat group repetitions
6th MN: 73	24	18	
7th MN: ___			7

MORE PROBLEMS FOR A REPETITION NUMBERS TABLE:

Meeting # (MN) for 3&4-pulse beat groups	3-pulse beat group repetitions	4-pulse beat group repetitions	12-pulse beat group repetitions
10th MN:___			
11^{th} MN: ___			
12th MN:-___			

ANOTHER PROBLEM:

Meeting # (MN) for 3&4-pulse beat groups	3-pulse beat group repetitions	4-pulse beat group repetitions	12-pulse beat group repetitions
100^{th} MN: __			

AN EXTRA CHALLENGE:
How long (in minutes, in seconds) do you think that it would take to play the 3-pulse beat group and the 4-pulse beat group up to the 100th Meeting Number? Explain how you arrived at your answer.

(**Hint**: Try playing up to the 1st or 2nd Meeting Number and find out how long that takes.)

Which pulse beat groups are being shown in the charts below?

Pulse Count	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
___-PBG	X						X						X		
___-PBG	X							X							X

Count	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
___-PBG				X						X					
___-PBG							X							X	

Count	31	32	3	34	35	36	37	38	39	40	41	42	43	44	45
___-PBG	X						X						X		
___-PBG						X							X		

What is the 1st Meeting Number of these pulse beat groups? _____

What Meeting Number Pulse Beat Group do they form? ___-PBG

Lesson 4 Vocabulary

Write down the new words that you learned in this lesson. Use each of the new words in a sentence.

LESSON 4
MORE EXPERIMENTS WITH PULSE BEAT GROUPS

LESSON OVERVIEW: In this lesson, we will conduct another experiment to learn more about what happens when you play a 3-pulse group together with a 4-pulse beat group. In Lesson 3, we learned that when you begin a 3-pulse beat group and a 4-pulse beat group together, they eventually meet again. In this lesson, we will find out how many pulse beats go by when a 3-pulse beat group and a 4-pulse beat group meet again.

Individual Work: Make a conjecture about when you think a 3-pulse beat group and a 4-pulse beat group will meet again, after beginning together on the starting (first) pulse beat.

Conjecture **--**
After starting together on pulse beat number 1, the 3-PBG and the 4-PBG will meet again for the first time on pulse beat number _____.

My reason(s) for making my conjecture are:
(You may also include drawings and/or diagrams to back up your conjecture.)

Class Work: We will try our experiment with pulse and pulse beat groups again. This time, we will add someone to count the pulse beats higher and higher. The experiment now requires the following:

(1) someone to play a pulse;

(2) someone to play a 3-Pulse Beat Group;

(3) someone to count a 3-Pulse Beat Group out loud to verify accuracy;

(4) someone to play a 4-Pulse Beat Group;

(5) someone to count a 4-Pulse Beat Group out loud to verify accuracy;

(6) someone to count the pulse higher and higher out loud, and;

(7) the rest of the class to determine what happens, and when.

Individual Work:

Experiment Result
After starting together on pulse beat number 1, the 3-PBG and the 4-PBG met for the first time on pulse beat number _____ .

Were you surprised by this result? _____

Why or why not?

Class Discussion: Discuss the result of the experiment and any ways that you and your classmates were surprised by what happened. Take notes in the space below.

Class Work: Use the table below to record the pulse beat number (pulse count) for the "Meeting Numbers" of a 3-pulse beat group and a 4-pulse beat group. A Meeting Number is when pulse beat groups sound on their "one" at the same time. Try the experiment from the last page again and record the information in the table below:

Meeting Numbers (MN's) for a 3-Pulse Beat Group and a 4-Pulse Beat Group:

	Pulse Beat #
Start #	1
1st MN	
2nd MN	
3rd MN	
4th MN	
5th MN	

Individual Work: Use the charts on the next page to show where the first five Meeting Numbers occur for a 3-PBG and a 4-PBG. Make an "X" in the spaces in the bottom rows where "X"s for the 3-PBG and the 4-PBG are lined up one on top of the other. (For example, the first place that this occurs is at the Start Number -- 1. An "X" has already been placed in the bottom row to show this "lining up".)

___-PBG and ___-PBG Meeting Number Chart

Pulse Count	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
___-PBG	X																								
___-PBG	X																								
MN-PBG	X																								

Pulse Count	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
___-PBG																									
___-PBG																									
MN-PBG																									

Pulse Count	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
___-PBG																									
___-PBG																									
MN-PBG																									

Team Work:
What Meeting Numbers did you find?

Start # -- ____ , 1st MN -- ____ , 2nd MN -- ____ , 3rd MN -- ____ , 4th MN -- ____ , 5th MN -- ____

How do the meeting numbers that you found using the chart compare with those that the class found by doing the meeting number experiment? (Were they the same or different?)

What size pulse beat group do the meeting numbers for a __-PBG and a __-PBG form? A _____ PBG

In other words:

If you count "one" beginning with the Start Number, and

Continue to count higher and higher until you reach the 1st Meeting Number when you count "one" again, and

Continue to count higher and higher until you reach the 2nd Meeting Number, and so on,

What size pulse beat group would you be counting?

What strategy did you use to come up with this answer?

Meeting Numbers (MN's) for a __-Pulse Beat Group and a __-Pulse Beat Group:

Start #	1
1st MN	
2nd MN	
3rd MN	
4th MN	
5th MN	

One strategy for figuring out the Meeting Number Pulse Beat Group (MN-PBG) is to use the chart on page 41 to count the pulse beat group formed by the Xs in the bottom row.

Note to team:

Your team should count out the pulse beat group formed in the bottom row.

Begin by counting "one" on the Start Number.

Count higher and higher until you come to the next Meeting Number, when you count "one" again, and so on.

In Lesson 4, we found the difference of consecutive Meeting Numbers to figure out the Meeting Number-Pulse Beat Group (MN-PBG). Try this method to find the MN-PBG for the pulse beat groups that you chose for the meeting number experiment in this lesson.

The 1st Meeting Number (___) minus the Start Number (1) is equal to ___ .

OR

The 2nd Meeting Number (___) minus the 1st Meeting Number (___) is equal to ___ .

OR

The 3rd Meeting Number (___) minus the 2nd Meeting Number (___) is equal to ___ .

What is your prediction for the difference between the 27th Meeting Number and the 26th Meeting Number? ____

Team Work:

Make Unifix™ cube models of:

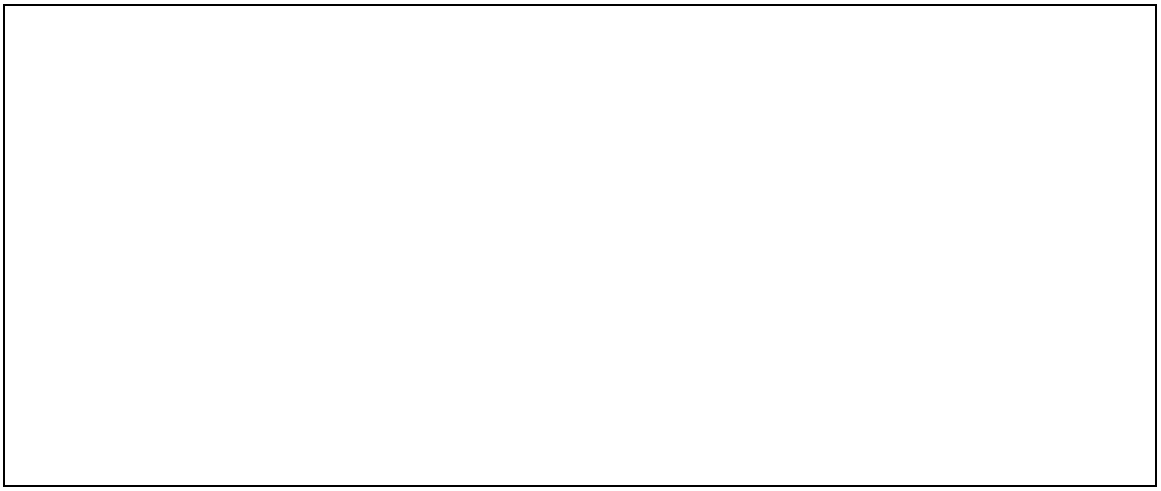
- The two different pulse beat groups that your team chose
-
- The Meeting Number Pulse Beat Group formed by the meeting numbers of your team's combination of pulse beat groups.

Make enough copies of the two different pulse beat groups that you chose for each of them to fit evenly into the Meeting Number Pulse Beat Group.

Draw a picture of your Unifix cube models of a ___-Pulse Beat Group, and a ___-Pulse Beat Group (the two different pulse beat groups that your team chose), and the new ___-Pulse Beat Group (your team's MN-PBG) that was created.

Your picture should show how many times each of the pulse beat groups that your team chose fit evenly into the new, Meeting Number Pulse Beat Group.

__-PBG, __-PBG, and __-PBG



Team Work: Complete the table below for the first five meeting numbers. Your will be using information about:

- The number of repetitions of a __-pulse beat group and a __-pulse beat group (both of the pulse beat groups that you chose), and;
- The number of repetitions of a __ -pulse beat group (the pulse group formed by the meeting numbers of pulse beat groups you chose).

REPETITION NUMBERS TABLE

Meeting #'s (MN's) for _&_-PBGs	__-PBG repetitions	__-PBG repetitions	(MN-PBG) __-PBG repetitions
1st MN: _____			
2nd MN: ____			
3rd MN: _____			
4th MN: _____			
5th MN: _____			

Team Discussion: What patterns can you find in your team's Repetition Numbers Table? Use the space below to write notes.

Class Work: Teams demonstrate the meeting number experiments that they did with their combinations of pulse beat groups.

Teams present the information in the various charts that they filled out and discuss the patterns that they found.

As teams report out, fill in the table below. Record the combination of pulse beat groups that teams used and the meeting number pulse beat group that each team found for its combination.

Meeting Number Pulse Beat Groups for Team Combinations of Pulse Beat Groups

Team Name	___-PBG	___-PBG	MN-PBG

Team Work: Complete the Repetitions Numbers Tables and the other problems below. The teacher will give you extra paper to do any calculations, draw any pictures or make any charts. Show all of your work. Each person on your team will be responsible for explaining your answers and how you came up with them.

Meeting #'s (MN's) for __&__-PBGs	__-pulse beat group repetitions	__-pulse beat group repetitions	(MN-PBG) 12-pulse beat group repetitions
1st MN: ___	3	2	1

Meeting # (MN) for 9&12-pulse beat groups	9-pulse beat group repetitions	12-pulse beat group repetitions	(MN-PBG) __-pulse beat group repetitions
1st MN: ___	4	3	1
2nd MN:___			2
10th MN:___			

Which pulse beat groups are being shown in the charts below?

Count	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
___-PBG	X					X					X				
___-PBG	X			X			X			X			X		

Count	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
___-PBG	X					X					X				
___-PBG	X			X			X			X			X		

Count	31	32	3	34	35	36	37	38	39	40	41	42	43	44	45
___-PBG	X					X					X				
___-PBG	X			X			X			X			X		

What Meeting Number Pulse Beat Group do they form? ___-PBG

Teams play/clap these pulse beat groups—one at a time (one PBG at a time) and both PBGs together.

Make a Repetition Numbers Table for these pulse beat groups and the MN-PBG that they form. Show the repetition numbers for the first five Meeting Numbers.

Lesson 5 Vocabulary

Write down the new words that you learned in this lesson. Use each of the new words in a sentence.

LESSON 5
EXPERIMENTING WITH NEW PULSE BEAT GROUPS

LESSON OVERVIEW: In this lesson, we will continue our meeting number experiments by working with different pulse beat groups. Teams will choose their own combinations of pulse beat groups.

Team Work: Teams will do the meeting number experiment again. This time, choose combinations of pulse beat group sizes that the class has not used yet. (You can choose a 5-PBG, 6-PBG, 7-PBG, 8-PBG, etc., to go with another PBG.)

Our team chose a ___-pulse beat group and a ___-pulse beat group.

Note to teacher(s) and students: Be sure that each team has a different combination of pulse beat groups.

The experiment requires the following:

(1) a person to play a pulse and count higher and higher;

(2) a person to play a __ - pulse beat group;

(3) a person to play a __ - pulse beat group;

(4) a person to observe and record meeting number information in the meeting number chart;

Predicting Meeting Numbers: Before you do the experiment, make predictions for the first five meeting numbers for the pulse beat groups you chose.

Write your predictions in the "Predicted" column of the Meeting Numbers Chart on the next page.

You will fill in the "Actual" column when you do the experiment.

Use the space below to figure out your predictions.

Team Name: _____

Meeting Numbers (MN's) Chart for a __-pulse beat group and a __- pulse beat group:

	Predicted	Actual
1st MN		
2nd MN		
3rd MN		
4th MN		
5th MN		

Now do the Meeting Number experiment with the combination of pulse beat groups that you chose and fill in the chart with the actual meeting numbers that you find.

Team Work: Use the charts on the next page to show where the first five Meeting Numbers occur for a __-PBG and a __-PBG. Make an "X" in the spaces in the bottom row where "X"s for the ___-PBG and the ___-PBG are lined up one on top of the other. (For example, the first place that this occurs is at the Start Number -- 1. An "X" has already been placed in the bottom row to show this "lining up".)

LESSON 6
FINDING MEETING NUMBERS AND FINDING LEAST COMMON MULTIPLES

LESSON OVERVIEW: In this lesson, we will learn about another way to find meeting numbers and meeting number pulse beat groups.

MEETING NUMBER ACTIVITY REVIEW

Thus far, we used a few different ways to find meeting numbers and meeting number pulse beat groups for different combinations of pulse beat groups.

We found meeting numbers:

- through sound: by actually playing two different pulse beat groups together and listening for when the "one" counts meet.

- through sight: by using charts to show what happens when you play two different pulse beat groups together and looking for the places where the "one" counts meet.

We found meeting number pulse beat groups:

- by counting "one" beginning with the Start Number and continuing to count higher and higher until we got to the 1st Meeting Number where we counted "one" again and continued to count higher and higher until we got to the 2nd Meeting Number, where we counted "one" again, and so on.

- by using Meeting Number Tables and taking the difference of consecutive meeting numbers.
  For example: the difference between the 1st Meeting Number and the Start Number, which is the same as:
  the 1st MN minus the Start Number;
  the 2nd MN minus the 1st MN; the 3rd MN minus the 2nd MN, and so on.

Class Work: Give specific examples from the work in earlier lessons that show the different ways that we used to find meeting numbers and meeting number pulse beat groups. Be sure to tell us which PBGs were used in your examples.

Class Work: Play a 3-pulse beat group and then fill in the table below for the number of pulse beats contained in different complete repetitions of a 3-PBG.

Pulse Beat Totals
3-PBG

# of 3-PBG Repetitions	Total # of Pulse Beats
1	
2	
3	
4	
5	
6	
7	
8	

Mathematicians have a name for the numbers in the column for total numbers of pulse beats for repetitions of the 3-PBG. These numbers are called **multiples** of 3, because they can be figured out by multiplying 3 by a **whole number**, such as 1, 2, 3, etc..

List the first eight **multiples** of 3: _____

Play a 4-PBG and then fill in the table below for the number of pulse beats contained in complete repetitions of a 4-PBG.

Pulse Beat Totals
4-PBG

# of 4-PBG Repetitions	Total # of Pulse Beats
1	
2	
3	
4	
5	
6	
7	
8	

List the first eight **multiples** of 4: _____

Play a 3-PBG and a 4-PBG together. Then use the information from the tables for pulse beat totals for the 3-PBG and pulse beat totals for the 4-PBG to fill in the table below

Pulse Beat Totals
3-PBG and 4-PBG

# of Repetitions	Total # of Pulse Beats 3-PBG	Total # of Pulse Beats 4-PBG
1		
2		
3		
4		
5		
6		
7		
8		

Circle the **multiples** that are shared by both the 3-PBG and the 4-PBG columns. In other words, circle the numbers that appear in both the 3-PBG and the 4-PBG columns. Mathematicians describe numbers that appear in both columns as "**common"** numbers. (That is, both columns have a number in common.)

What numbers did you circle?

_____

What number is the smallest multiple common to both the 3-PBG and the 4-PBG columns – in other words what is the smallest number that appears in both columns? _____

How does this number relate to the size of the Meeting Number Pulse Beat Group for a 3-PBG and a 4-PBG?

Class Discussion: Mathematicians have a name for a smallest common multiple. They call it a **Least Common Multiple (LCM)**. Why do you think that this kind of number is called a "least common multiple"?

Team Work: Complete the table and answer the questions below for the pulse beat groups that your team chose to work with in Lesson 5.

Pulse Beat Totals
___-PBG and ___-PBG

# of Repetitions	Total # Pulse Beats –– ___-PBG	Total # of Pulse Beats –– ___-PBG
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Circle the numbers that are common to both of the columns for the total number of pulse beats for your team's pulse beat groups.

What numbers did you circle?

_____

What number is the smallest number that both of these columns share? In other words, what number is the least common multiple?

_____

How does this least common multiple relate to the size of the Meeting Number Pulse Beat Group for your team's pulse beat groups?

Class Work: Teams report out.

Class Discussion: We have just used a strategy for finding out the size of the Meeting Number Pulse Beat Group, that is to:

- Make lists of the number of pulse beats for the repeating PBGs, and,
- Identify the smallest number that appears in both lists.

How can you use the size of the MN-PBG to figure out the 1st MN?

(Give examples.)

How can you use the size of the MN-PBG to figure out the 2nd MN? …the 10th MN?

(Give examples.)

Class Discussion: Teams will use a "Multiples Table" on the next page to figure out the sizes of the Meeting Number Pulse Beat Group for different combinations of pulse beat groups.

The number of pulse beats for repetitions of a particular pulse beat group is the same as the multiples of that **Size** pulse beat group.

For example:

The total number of pulse beats for complete repetitions of a 3-Pulse Beat Group turned out to be the multiples of 3.

The total number of pulse beats for complete repetitions of a 4-Pulse Beat Group turned out to be the multiples of 4.

<u>The Size of a pulse beat group is the same as the Number of Pulse Beats in that Group.</u>

In the Multiples Table on the next page, the numbers in the top row (**1**, **2**, **3**, **4**, **5**, **6**, **7**, **8**, **9**, **10**, **11**, **12**) represent the **sizes**, or **number of pulse beats per group**, for different pulse beat groups.

The numbers in the left column (**1**, **2**, **3**, **4**, **5**, **6**, **7**, **8**, **9**, **10**, **11**, **12**) represent the number of **repetitions of the pulse beat group**.

Team Work: Use the Multiples Table below to figure out the sizes (number of pulse beats per group) of the Meeting Number Pulse Beat Groups for:

- A 6-PBG and an 8-PBG
- A 3-PBG and a 9-PBG
- An 11-PBG and a 12-PBG
- A 6-PBG and a 4-PBG

Use the size of the Meeting Number Pulse Beat Group to determine the 1st, 2nd, and 10th Meeting Numbers for each pair of pulse beat groups. Put your answers in the Multiples Table, below.

Multiples Table
(for Repetitions/Pulse Beats Per Group)

		Pulse Beats Per Group											
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
R	**1**	1	2	3	4	5	6	7	8	9	10	11	12
E	**2**	2	4	6	8	10	12	14	16	18	20	22	24
P	**3**	3	6	9	12	15	18	21	24	27	30	33	36
I	**4**	4	8	12	16	20	24	28	32	36	40	44	48
T	**5**	5	10	15	20	25	30	35	40	45	50	55	60
I	**6**	6	12	18	24	30	36	42	48	54	60	66	72
T	**7**	7	14	21	28	35	42	49	56	63	70	77	84
I	**8**	8	16	24	32	40	48	56	64	72	80	88	96
O	**9**	9	18	27	36	45	54	63	72	81	90	99	108
N	**10**	10	20	30	40	50	60	70	80	90	100	110	120
S	**11**	11	22	33	44	55	66	77	88	99	110	121	132
	12	12	24	36	48	60	72	84	96	108	120	132	144

Pulse Beat Group A	Pulse Beat Group B	MN-PBG	1st MN	2nd MN	10th MN
6-PBG	8-PBG				
3-PBG	9-PBG				
11-PBG	12-PBG				
6-PBG	4-PBG				

Describe how you used the size of the Meeting Number Pulse Beat Group to figure out the 1st MN, 2nd MN, and 10th MN for each pair of pulse beat groups.

Clap each of the pairs of pulse beat groups along with a pulse.

Class Work: Teams report out and the class discusses the results.

Lesson 6 Vocabulary

Write down the new words that you learned in this lesson. Use each of the new words in a sentence.

LESSON 7
PULSE BEAT GROUPS AND FACTORS

Class Discussion: In this lesson, we will identify and play the pulse beat groups that fit evenly into a particular pulse beat group. We will use these investigations to learn about some other ideas that mathematicians use in their work.

Individual Work: Copy your drawing from page 27 in Lesson 4 that shows how many 3-PBGs and how many 4-PBGs fit evenly into a 12-PBG. (Your drawing of your Unifix™ cube models.)

12-PBG, 3-PBG, and 4-PBG

Individual Work: Find all of the other pulse beat groups that fit evenly into a 12-pulse beat group.

Use the space below to draw a picture of a 12-pulse beat group along with:

- a 3-Pulse Beat Group,
- a 4-Pulse Beat Group, and,
- all of the other pulse beat groups that fit evenly into a 12-Pulse Beat Group.

Draw and label each of the pulse beat groups you found and show how many times each of them fits into a 12-PBG.
Note: You can create Unifix cube models if you would like.

The PBGs that Fit with a 12-PBG

Individual Work: Complete the chart below. Be sure to show the pulse beat groups that you found and the number of times (number of repetitions) each pulse beat group fits evenly with a 12-PBG.

Pulse Beat Groups that Evenly Fit into a 12-PBG

PBG	Pulse Beats Per Group	Number of Repetitions	Total Number of Pulse Beats
3-PBG	3	4	12
4-PBG	4	3	12
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			

How many 3-PBGs fit evenly with a 12-PBG?

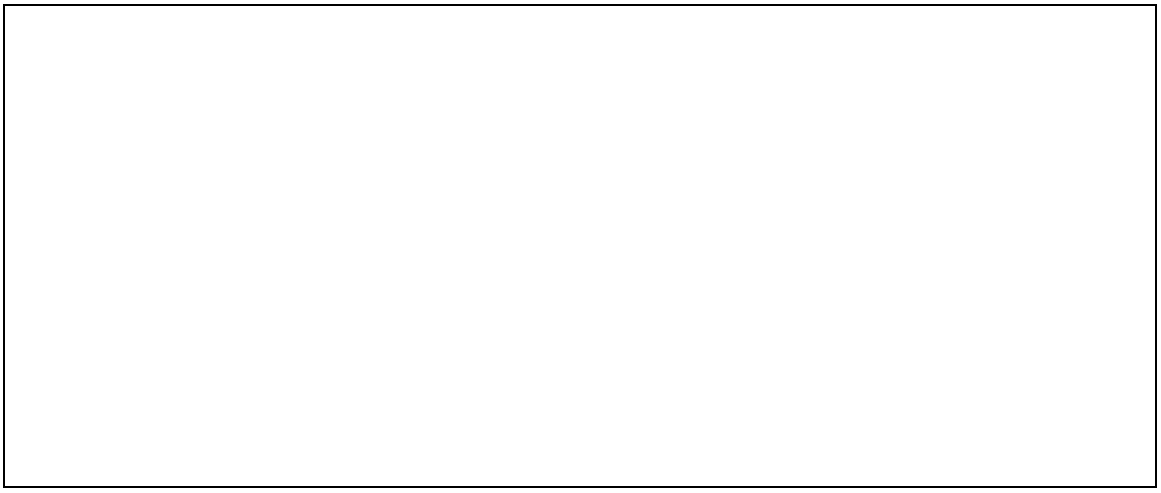
Class Work: Discuss the results of your investigations.

Play a 12-PBG along with all of the pulse beat groups that fit evenly into it. Choose players and counters for the 12-PBG and each of the pulse beat groups that you found.

How can you know if you played the parts correctly?

Team Work: Copy your drawing from page 42 in Lesson 5 that shows how many times the pulse beat groups that your team chose fit into your team's Meeting Number Pulse Beat Group.

__-PBG (MN-PBG), __-PBG, and __-PBG



Team Work: Find all of the other pulse beat groups that fit evenly with your team's Meeting Number Pulse Beat Group (a ___-pulse beat group).

Use the space below to draw a picture of a __-pulse beat group (your Meeting Number Pulse Beat Group) along with:

- a __-PBG,
- a __-PBG (the pulse beat groups that your team chose in Lesson 5), and,
- all of the other pulse beat groups that fit evenly with your Meeting Number Pulse Beat Group.

Draw and label each of the pulse beat groups you found and show how many times each of them fits with your MN-PBG.

The PBGs that Fits evenly with a ___-PBG

Team Work: Complete the table below that shows the pulse beat groups that you found and the number of times, or number of repetitions, that they fit.

Pulse Beat Groups that Evenly Fit Your Team's MN-PBG
___-PBG

PBG	Pulse Beats Per Group	Number of Repetitions	Total Number of Pulse Beats
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			

Team Music: Practice playing your MN-PBG along with all of the pulse beat groups that fit.

Class Work: Teams share what they found about the pulse beat groups that fit evenly into their Meeting Number Pulse Beat Groups. Teams play their Meeting Number Pulse Beat Groups along with all the other pulse beat groups that fit.

Class Discussion: When you found all of the different pulse beat groups that fit into your Meeting Number Pulse Beat Groups, you were working with an idea that mathematicians call **factors**.

The **factors** of a number are the numbers that divide evenly into that number.

For example, when we worked with a 12-PBG (the Meeting Number Pulse Beat Group for a 3-PBG and a 4-PBG), we found the following pulse beat groups that fit.

Pulse Beat Groups that Evenly Fit a 12-PBG

PBG	Pulse Beats Per Group	Number of Repetitions	Total Number of Pulse Beats
1-PBG	1	12	12
2-PBG	2	6	12
3-PBG	3	4	12
4-PBG	4	3	12
6-PBG	6	2	12
12-PBG	12	1	12

The pulse beat groups that fit evenly with a 12-PBG are also the **factors** of the number 12.

Notice that the same numerals are in the Pulse Beats Per Group column and in the Numbers of Repetitions column. We can use either column to find the **factors** of 12. Why do you think that the same numerals are in both columns?

Using the table above, what are the **factors** of 12? (Write the **factors** in the space below.)

MAKING NUMBER SENTENCES

Mathematicians often use ways of writing that are different from everyday language. We can make some different **number sentences** from the information contained in the table on the previous page.

For example: a 3-PBG fits into a 12-PBG four times.

We can express this idea using the following **number sentence**:

$$12 \div 3 = 4$$

In this number sentence, "12" represents the number of pulse beats in a 12-PBG. The "3" represents the number of pulse beats in a 3-PBG. The "4" represents the number of times (or repetitions) a 3-PBG can fit into a 12-PBG.

We can also say and write:

Four repetitions of a 3-PBG fit evenly into a 12 PBG.

$$4 \times 3 = 12$$

Class Discussion:

How do these two number sentences ($12 \div 3 = 4$ and $4 \times 3 = 12$) relate to each other?

Use the table on the previous page to make different number sentences that describe relationships between a 12-PBG and the pulse beat groups that fit evenly into a 12-PBG. Use the space below to write down the number sentences that you and your classmates create.

Class Discussion:

In Lesson 6, we learned about **multiples**. In this lesson, we have begun to study **factors**.

What are the ways in which **multiples** and **factors** are different? How are **multiples** and **factors** alike?

As the class discusses these questions, fill in the chart below.

Comparing Multiples and Factors	
Ways in which **multiples** and **factors** are different	Ways in which **multiples** and **factors** are alike

Team Work:

Complete the table and answer the questions below.

Pulse Beat Groups that Evenly Fit an 18-PBG

PBG	Pulse Beats Per Group	Number of Repetitions	Total Number of Pulse Beats
1-PBG	1		18
__-PBG	2		
__-PBG		6	
__-PBG		3	
__-PBG			
18-PBG	18	1	18

Use the table above to make 6 or more number sentences that express the relationship between an 18-PBG and the pulse beat groups that fit evenly into an 18-PBG.

Choose a different pulse beat group and complete the table below

Pulse Beat Groups that Evenly Fit A ___-PBG

PBG	Pulse Beats Per Group	Number of Repetitions	Total Number of Pulse Beats
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			

Use the table above to make 6 or more number sentences that express the relationship between an __-PBG (the pulse beat group that your team chose) and the pulse beat groups that fit.

Lesson 7 Vocabulary

Write down the new words that you learned in this lesson. Use each of the new words in a sentence.

Class Work: Discuss and demonstrate the results of the team investigations. As teams report out, fill in the "Pulse Beat Group Factors" below.

Pulse Beat Group Factors

	Pulse Beats Per Group														
F	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
A	1	1	1												
C		2	3												
T															
O															
R															
S															

Class Discussion: Use the space below to take notes from your discussion about pulse beat group factors.

What patterns do you see in the "Pulse Beat Group Factors" table on the previous page?

Class Discussion: There are many patterns that can be found as we work with pulse beat group factors.

Among the patterns that you might have observed:

All of the pulse beat groups have a 1-PBG as a factor. In other words, "1" is a factor of all of the size numbers (1, 2, 3, …15) that we studied.

Some of the pulse beat groups have only a 1-PBG and themselves as factors. Mathematicians have a name for a number that has only two factors – itself and 1.

This kind of number is called a **prime number**.*

Some of the pulse beat groups have other pulse beat group factors in addition to a 1-PBG and themselves. Mathematicians have a name for a number that has more than two factors.

This kind of number is called a **composite number**.

A 2-PBG fits with all of the **even number**** sized pulse beat groups (such as a 2-PBG, 4-PBG, 6-PBG, etc.). This is a way of showing that 2 is a factor of every even number. Remember, all even numbers have a 0, 2, 4, 6, or 8 in the ones place.

We will be working more with **factors**, **prime numbers**, and **composite numbers** in the lessons that follow.

* The number "1" is not a **prime number** because it only has one factor.

** Even numbers other than the number 2 are all **composite numbers.**

FINDING PRIME NUMBERS

Individual Work:

Using the Number Chart on the next page, follow these steps in order to find the **prime numbers** that exist between 1 and 100.

- Cross out "1".
- Circle "2" and cross out every other number after 2. (In other words, cross out all of the **multiples** of 2 except for 2 itself.)
- Circle "3" and cross out every third number after 3. (In other words, cross out all of the multiples of 3 except for 3 itself.)
- Circle "5" and cross out every fifth number after 5. (In other words, cross out all of the multiples of 5 except for 5 itself.)
- Circle "7" and cross out every seventh number after 7. (In other words, cross out all of the multiples of 7 except for 7 itself.)
- Circle all of the remaining numbers that haven't been crossed out or circled.

Number Chart
1 - 100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Individual Work: Complete the table below. Make lists of the numbers that you circled and numbers that you crossed out.

Numbers Table

Numbers That You Circled	Numbers That You Crossed Out

Class Discussion:

Share the numbers that you circled in the chart. These numbers are the **prime numbers** between 1 and 100.

What **prime numbers** exist among the pulse beat groups that we studied in this lesson?

The numbers that you crossed out, except for "1", are all **composite numbers**.

What **composite numbers** can be found among the pulse beat groups that we studied in this lesson?

Why did we skip the step of circling 4 and crossing out the multiples of 4? …

Answer this same question for 6, 8, and 9.

Why did we still need to circle 5 and eliminate the multiples of 5?

Why did we need to circle 7 and eliminate the multiples of 7?

Why was it unnecessary to cross out the multiples of numbers higher than 10?

Lesson 8 Vocabulary

Write down the new words that you learned in this lesson. Use each of the new words in a sentence.

LESSON 8
MORE PULSE BEAT GROUPS AND FACTORS

LESSON OVERVIEW: In this lesson, we will continue to study **factors** by finding the pulse beat groups that fit evenly into a particular pulse beat group.

Class Discussion:

What are all of the pulse beat groups that fit into a 1-PBG? …a 2-PBG? …a 3-PBG?

As you discuss this question, play a 1-PBG, a 2-PBG, and a 3-PBG along with the pulse beat groups that fit.

You will work in your teams to identify all of the different pulse beat groups that fit into:

> a 4-PBG, a 5-PBG, a 6-PBG, a 7-PBG, an 8-PBG, a 9-PBG, a 10-PBG, an 11-PBG, a 12-PBG, a 13-PBG, a 14-PBG, and a 15-PBG

The teacher will assign pulse beat groups for each team to work on so that all of the above pulse beat groups are covered.

Team Work:

For each pulse beat group that your team is assigned, fill in a "Pulse Beat Group Factors" chart that shows all of the pulse beat groups that fit and how many times they fit.

After you complete the charts, practice playing your assigned pulse beat groups along with all of the pulse beat groups that fit.

Team Name: _____

We were assigned: a ___-PBG, a ___-PBG, a ___-PBG, a ___-PBG

Pulse Beat Group Factors of a ___-PBG

PBG	Pulse Beats Per Group	Number of Repetitions	Total Number of Pulse Beats
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			

Pulse Beat Group Factors of a ___-PBG

PBG	Pulse Beats Per Group	Number of Repetitions	Total Number of Pulse Beats
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			

Team Name: _____

Pulse Beat Group Factors of a ___-PBG

PBG	Pulse Beats Per Group	Number of Repetitions	Total Number of Pulse Beats
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			

Pulse Beat Group Factors of a ___-PBG

PBG	Pulse Beats Per Group	Number of Repetitions	Total Number of Pulse Beats
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			
__-PBG			

Team Music
Play your assigned pulse beat groups along with the pulse beat groups that fit.

Team Work: Use the "Pulse Beat Group Factors" table below to find the **greatest common factors** for the following pairs of numbers:

4 and 6; 5 and 7; 8 and 10; 4 and 12; 5 and 15; 13 and 14; 2 and 8; 6 and 9

Table of Pulse Beat Group Factors

	Pulse Beats Per Group														
F	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C		2	3	2	5	2	7	2	3	2	11	2	13	2	3
T				4		3		4	9	5		3		7	5
O						6		8		10		4		14	15
R												6			
S												12			

Class Discussion:

Discuss the following questions:

What is the greatest common factor when both numbers in a pair are prime numbers? (for example, 5 and 7) Give other examples.

What is the greatest common factor when one number is a factor of the other? (for example, 3 and 9) Give other examples.

What do you notice about pairs of numbers when "1" is the greatest common factor?

Lesson 9 Vocabulary

Write down the new words that you learned in this lesson. Use each of the new words in a sentence.

LESSON 9
GREATEST COMMON FACTORS

LESSON OVERVIEW: In this lesson, we will work with pairs of pulse beat groups to learn about a mathematics concept called "Greatest Common Factor."

Class Work: Complete the Factor Table for a 12-PBG and a 15-PBG.

Factor Table – 12-PBG and 15-PBG

Pulse Beats Per Group	12	15
Factors		

Circle the factors that are common to both lists.

What factors did you circle?

What is the largest factor that is common to both lists? ____

Class Discussion: Mathematicians have a name for this kind of number. It is called a **Greatest Common Factor (GCF)**. Why do you think that this kind of number is called a "greatest common factor"?

Team Work: Complete the table and answer the questions below for the pulse beat groups that your team chose to work with in Lesson 13.

Factor Table – ___-PBG and ___-PBG

Pulse Beats Per Group	___	___
Factors		

Circle the factors that are common to both lists.

What factors did you circle?

What is the largest factor that is common to both lists? ____

Class Work: Teams present their findings.

Multiples Table

		b														
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
a	**1**	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
	3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
	4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
	5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
	6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
	7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105
	8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120
	9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
	10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
	11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165
	12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
	13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195
	14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210
	15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225

Factors Table

a or b	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Factors	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		2	3	2	5	2	7	2	3	2	11	2	13	2	3
				4		3		4	9	5		3		7	5
						6		8		10		4		14	15
												6			
												12			

Team Work: Share your Individual Work. Answer the questions below.

What patterns do you see in the above table?

What connections do you see between the LCM and the GCF for pairs of numbers?

Class Work: Teams report out.

Team Work: Complete the table below. Although you will be working in teams, each team member is responsible for completing all of the work and for being able to explain how you came up with your solutions.

Pulse Beats Per Group **a-PBG**	**Pulse Beats Per Groups** **b-PBG**	**a x b**	**Least Common Multiple of a and b**	**Greatest Common Factor of a and b**	**LCM of a and b x GCF of a and b**
2	4				8
9	6	54		3	
5		35			
	6			3	18
9				3	108

Based upon the work that you have done in completing the tables in this lesson, complete the sentences below.

a x b = GCF of a and b x _____

a x b = _____ x LCM of a and b

<u>Extra Credit</u>
GCF of a and b = a x b divided by _____

LCM of a and b = a x b divided by _____

Lesson 10 Vocabulary

Write down the new words that you learned in this lesson. Use each of the new words in a sentence.

LESSON 10
CONNECTIONS BETWEEN GREATEST COMMON FACTORS AND LEAST COMMON MULTIPLES

LESSON OVERVIEW: In this lesson, we will look at some of the connections between **Greatest Common Factors** (GCF) and **Least Common Multiples** (LCM).

We will also begin to talk about "variables", an idea that we use in our everyday lives and that is an important part of doing mathematics.

Class Discussion: In order to complete the table on the next page, we will be using an idea called **variables**. Actually, we have been using this idea in earlier lessons.

Two examples of **variables** that we have used so far are:

- Name:_____
- ___-PBG

What does the "_____" mean in "Name:_____"?

What does the "___" mean in "___-PBG"?

Why do you think that the "___" and the "_____" might be called **variables**?

Think of other examples of **variables** and write them down in the space below.

In the table below, you will notice in the column headings that the letters "a" and "b" are used in places, such as "a-PBG" or "a x b", where numbers or "____"s were used in earlier lessons.

The "a" and "b" are being used as **variables**. The numerical **values** of "a" and "b" **vary**, or change, from one example to the next.

What are some of the different values that "a" and "b" have in the table below?
Individual Work: Complete the table below. The first two rows have already been done. In the last two rows, you will need to choose the values of "a" and "b" yourselves. Choose numbers from 2 up to 15.

You may use the "Multiples Table" and "Factors Table" on the next two pages to help complete the work.

Pulse Beats Per Group **a-PBG**	**Pulse Beats Per Groups** **b-PBG**	**a x b =**	**Least Common Multiple of a and b**	**Greatest Common Factor of a and b**	**LCM of a and b x GCF of a and b**
3	4	12	12	1	12
6	8	48	24	2	48
12	15				
4	6				
12	8				

LESSON 11
MAKING COMPARISONS BY SUBTRACTION AND RATIO

LESSON OVERVIEW: In this lesson, we will study two different ways of comparing.

Class Discussion
What does it mean to "Compare"? Give examples of times when you have made comparisons. Use the space below to take notes.

TWO KINDS OF COMPARISON

You will use two methods to Compare Features of the Meeting Number Activity:

(1) Subtraction Comparison

(2) Ratio Comparison

Mathematicians use questions to help them compare features of things.

SUBTRACTION COMPARISON

Subtraction Comparisons answer the following types of questions:

"How much _____
(larger/smaller, taller/shorter, hotter/colder, etc.)?"

Examples

How much taller (shorter) is Kareem than LaTeesha?

How much hotter is it in New Orleans than in Chicago?

How much larger is the number of pulse beats
in a 4-Pulse Beat Group than the number of pulse beats
in a 3-Pulse Beat Group?

For example, to make a **subtraction comparison** of the number of girl students compared to the number of boy students in a classroom with 12 girls and 10 boys, we would ask, in "feature talk,"

Feature Talk Question

What is the number of girl students **compared by subtraction** to the number of boy students?

Feature Talk Response

The number of girl students in the classroom **compared by subtraction** to the number of boy students is 2 students greater.

We can show this in a **number sentence**.

Number Sentence

$12 - 10 = 2$

We can also compare the number of boy students to the number of girl students in the class.

Comparing the number of boy students to the number of girl students in the class can be thought of as being the **opposite** of comparing the number of girl students to the number of boy students.

Why do you think that these comparisons can be called **opposites**?

Feature Talk Question

What is the number of boy students **compared by subtraction** to the number of girl students?

Feature Talk Response

The number of boy students in the classroom **compared by subtraction** to the number of girl students is 2 students less.

We can show this in a **number sentence**.
However, before writing the number sentence, we need a way to show that there are 2 students less.

"2 students less" is the **opposite** of "2 students greater."

Mathematicians show "2 less" in a number sentence by using a minus sign in front of the "2".

To show that the number of boy students compared by subtraction to the number of girl students is 2 students less, we can use the following number sentence:

Number Sentence

$$10 - 12 = -2$$

FINDING EQUIVALENT SUBTRACTION COMPARISONS

Individual Work: Make up 3 other classroom situations in which the number of girl students is 2 students greater than the number of boy students (or the number of boy students is 2 students less than the number of girl students).

Classroom 1
Number of girl students ____ , Number of boy students ____

Make diagrams that show that the number of girl students compared by subtraction to the number of boy students is 2 students greater and that the number of boy students compared by subtraction to the number of girl students is 2 students less.

Write a number sentence that shows that the number of girl students compared by subtraction to the number of boy students is 2 students greater.

Number Sentence

Write a number sentence that shows that the number of boy students compared by subtraction to the number of girl students is 2 students less.

Number Sentence

Classroom 2
Number of girl students ____ , Number of boy students ____

Make diagrams that show that the number of girl students compared by subtraction to the number of boy students is 2 students greater and that the number of boy students compared by subtraction to the number of girl students is 2 students less.

Write a number sentence that shows that the number of girl students compared by subtraction to the number of boy students is 2 students greater.

Number Sentence

Write a number sentence that shows that the number of boy students compared by subtraction to the number of girl students is 2 students less.

Number Sentence

Classroom 3
Number of girl students ____ , Number of boy students ____

Make diagrams that show that the number of girl students compared by subtraction to the number of boy students is 2 students greater and that the number of boy students compared by subtraction to the number of girl students is 2 students less.

Write a number sentence that shows that the number of girl students compared by subtraction to the number of boy students is 2 students greater.

Number Sentence

Write a number sentence that shows that the number of boy students compared by subtraction to the number of girl students is 2 students less.

Number Sentence

Class Discussion: Use the examples that you came up with in your Individual Work to discuss the following questions:

How are the subtraction comparisons for the classroom situations that you found **equivalent** (the same)?

How are they different?

Give examples of some of the **opposite** subtraction comparisons that you found.

What are other examples of situations in which you could use subtraction comparisons to compare things, people, or places?

RATIO COMPARISON

Class Discussion:
Ratio Comparisons answer the following types of questions:

For every _____ , how many _____ ?

Examples

For every 5 girls, how many boys?

For every 2 cups of honey, how many cups of whole wheat flour?

For every 1 teacher, how many students?

For every four 3-Pulse Beat Groups, how many 4-Pulse Beat Groups?

For example, to make a **ratio comparison** of the number of girl students compared to the number of boy students in a classroom with 12 girls and 10 boys, we would ask, in feature talk,

Feature Talk Question

What is the number of girl students **compared by ratio** to the number of boy students?

Mathematicians use several steps to answer this type of question.

A first response to the question is:

Feature Talk Response

The number of girl students in the classroom **compared by ratio** to the number of boy students is 12 to 10.

This statement is another way of saying: **for every 12 girl students, there are 10 boy students**.

TEAM WORK: Make a Unifix cube model that shows this comparison. Choose one color for girls and another color for boys.

REDUCING RATIO COMPARISONS TO LOWEST TERMS

Class Discussion:
After first making a ratio comparison with the original quantities (amounts), mathematicians next check for the lowest numbers that can preserve the relationship between the quantities (amounts) that are being compared. This process is called "**reducing"** or "**renaming"** ratio comparisons to "**lowest terms"**.

Reducing or renaming ratio comparisons to lowest terms is useful because it is often easier to visualize and do computations with smaller numbers. Reducing ratios also makes it easier to determine how various ratios are different and alike.

In order to reduce a ratio comparison to lowest terms:

- Find the greatest common factor of the two numbers being compared. (If the greatest common factor is 1, the ratio comparison is already in lowest terms.)
- Divide each of the numbers being compared by the greatest common factor.
- Rewrite the ratio comparison using the quotients from the divisions by the greatest common factor.

We will use our Unifix cube models to help us think about these operations.

The diagrams below can be used to represent a classroom with 12 girl students and 10 boy students, and are similar to your Unifix cube models.

(In the diagrams, "g" is a symbol for "girl student" and "b" is a symbol for "boy student".)

1	2	3	4	5	6	7	8	9	10	11	12
g	g	g	g	g	g	g	g	g	g	g	g

1	2	3	4	5	6	7	8	9	10
b	b	b	b	b	b	b	b	b	b

To reduce the ratio comparison "12 to 10" to lowest terms:

Find the greatest common factor of 12 and 10.
The greatest common factor of 12 and 10 is 2. (See the Factors Table on p. 92)

Divide the numbers being compared – 12 and 10 – by the greatest common factor.

$12 \div 2 = 6$ $10 \div 2 = 5$

Rewrite the ratio comparison – 12 to 10 – using the quotients from the division by the greatest common factor.
The ratio comparison of the number of girl students compared to the number of boy students is 6 to 5.

The diagrams below show the 12 girl students and the 10 boy students each divided into 2 equal groups. This is a way of representing the step in which we divided 12 and 10 by their greatest common factor, 2.

1	2	3	4	5	6
g	g	g	g	g	g

7	8	9	10	11	12
g	g	g	g	g	g

1	2	3	4	5
b	b	b	b	b

6	7	8	9	10
b	b	b	b	b

The diagrams show that in a classroom with 12 girl students and 10 boy students, **for every 6 girl students, there are 5 boy students**.

We can now answer the original question using the reduced ratio comparison that we found on the previous page.

Feature Talk Question

What is the number of girl students **compared by ratio** to the number of boy students?

Feature Talk Response

The number of girl students **compared by ratio** to the number of boy students is 6 to 5.

Mathematicians write ratio comparisons in different ways. Two ways of writing "6 to 5" using mathematical symbols are:

$$6 : 5$$

and

$$6/5.$$

Create a Unifix cube model of this reduced ratio comparison.

EQUIVALENT RATIO COMPARISONS

In the classroom example, we found that:

"For every 12 girl students, there are 10 boy students."

is **equivalent** to

"For every 6 girl students, there are 5 boy students."

This idea can be expressed in a number sentence:

$$12/10 = 6/5$$

The diagrams below show another example of a **ratio comparison** that is **equivalent** to 6/5.

1	2	3	4	5	6
g	g	g	g	g	g

7	8	9	10	11	12
g	g	g	g	g	g

1	2	3	4	5
b	b	b	b	b

6	7	8	9	10
b	b	b	b	b

13	14	15	16	17	18
g	g	g	g	g	g

11	12	13	14	15
b	b	b	b	b

What is the number of girl students in this example? _____
What is the number of boy students? _____

Write the number of girl students compared to the number of boy students as a **ratio**.

How is this ratio **equivalent** to 6/5?

Write a number sentence to show that this ratio is **equivalent** to 6/5.

Number Sentence

____ = 6/5

Individual Work: Use the worksheets on the next 3 pages to identify 3 other classroom situations in which there are 6 girl students for every 5 boy students. Make diagrams and write number sentences for each of these situations that show that the **ratio comparisons** are **equivalent** to 6/5.

Classroom 1
Number of girl students ____ , Number of boy students ____

Make diagrams that show that the number of girl students compared by ratio to the number of boy students is equivalent to 6/5.

Write a number sentence that shows that the number of girl students compared by ratio to the number of boy students is equivalent to 6/5.

Number Sentence

Optional activity: Create Unifix cube models for this new comparison

Classroom 2
Number of girl students ____ , Number of boy students ____

Make diagrams that show that the number of girl students compared by ratio to the number of boy students is equivalent to 6/5.

Write a number sentence that shows that the number of girl students compared by ratio to the number of boy students is equivalent to 6/5.

Number Sentence

Classroom 3
Number of girl students ____ , Number of boy students ____

Make diagrams that show that the number of girl students compared by ratio to the number of boy students is equivalent to 6/5.

Write a number sentence that shows that the number of girl students compared by ratio to the number of boy students is equivalent to 6/5.

Number Sentence

Class Discussion: Use the examples that you came up with in your Individual Work to discuss the following questions:

How are the ratio comparisons for the classroom situations that you found equivalent?

How are they different?

What strategies did you use to figure out classroom situations in which the ratio comparisons were equivalent to 6/5?

What are other examples of situations in which you could use ratio comparisons to compare things, people, or places?

LESSON SUMMARY

Class Discussion: In this lesson, you were introduced to two kinds of comparisons – **subtraction comparisons** and **ratio comparisons**. Each of these comparisons provide different types of information.

Subtraction comparisons answer questions such as "How much larger (or smaller)?"

Ratio comparisons answer questions such as "For every _____, how many _____?"

You also began to study **equivalent subtraction comparisons** and **equivalent ratio comparisons**.

Subtraction comparisons, such as 5 – 3 and 89 – 87, are said to be **equivalent** when the quantities being compared are greater than (or less than) by the same amount. For example 5 girl students are 2 students greater than 3 boy students and 89 girl students are 2 students greater than 87 boy students.

Ratio comparisons, such as 12/10 and 24/20, are **equivalent** when they can be **reduced** to the same ratio. Because 12/10 and 24/20 both reduce to the same ratio, 6/5, they are **equivalent**.

If the greatest common factor of the quantities being compared is "1", the ratio comparison is already reduced to lowest terms.

If you already know the ratio reduced to lowest terms, you can find equivalent ratios by multiplying both of the quantities being compared by the same number. For example to find other ratios equivalent to 6/5, you can multiply both 6 and 5 by 2, 3, 4, 5, …100, and so on. (Try some of these.)

Team Work: Complete the subtraction and ratio comparisons that follow. You will be working in teams, BUT: each student is responsible for being able to explain your work and solutions.

Use the worksheet on the next page to make a **subtraction comparison** of the number of girl students compared to the number of boy students in your classroom. Make diagrams and write a number sentence to show this subtraction comparison.

Our Classroom

Number of girl students ____ , Number of boy students ____

The number of girl students compared by subtraction to the number of boy students is ___ students greater/less.
(Fill in the blank) **(Circle one.)**

Make diagrams that show the number of girl students compared by subtraction to the number of boy students.

Write a number sentence that shows the number of girl students compared by subtraction to the number of boy students.

Number Sentence

Use the worksheets on the next 2 pages to identify 2 other classroom situations in which the **subtraction comparison** of the number of girl students compared to the number of boy students is equivalent to the subtraction comparison in your classroom.

Classroom 1
Number of girl students ____ , Number of boy students ____

The number of girl students compared by subtraction to the number of boy students is ___ students greater/less.
(Fill in the blank) **(Circle one.)**

Make diagrams that show the number of girl students compared by subtraction to the number of boy students.

Write a number sentence that shows the number of girl students compared by subtraction to the number of boy students.

Number Sentence

Classroom 2
Number of girl students ____ , Number of boy students ____

The number of girl students compared by subtraction to the number of boy students is ___ students greater/less.
(Fill in the blank) **(Circle one.)**

Make diagrams that show the number of girl students compared by subtraction to the number of boy students.

Write a number sentence that shows the number of girl students compared by subtraction to the number of boy students.

Number Sentence

Team Work: Make a **ratio comparison** of the number of girl students compared to the number of boy students in your classroom.

Our Classroom

Number of girl students ____ , Number of boy students ____

The number of girl students compared by ratio to the number of boy students is ____ .

Reduce the ratio comparison to lowest terms.

Find the greatest common factor (GCF) of ___ (the number of girl students) and ___ (the number of boy students).

GCF = ____

If the GCF is 1, then the ratio comparison is already in lowest terms.

If the GCF is not 1, divide each of the quantities being compared by the GCF.

Rewrite the ratio comparison using the quotients from the divisions by the greatest common factor.

The number of girl students compared by ratio to the number of boy students is ____ . (Write the ratio in lowest terms.)

Make diagrams that show the number of girl students compared by ratio to the number of boy students in your classroom. If you also reduced the original ratio comparison to an equivalent ratio in lowest terms, show this in your diagrams, as well.

Write a number sentence that shows that the number of girl students compared by ratio to the number of boy students is equivalent to the ratio reduced to lowest terms. (If the original ratio is already in lowest terms, your number sentence will show that the ratio is equivalent to itself.)

Number Sentence

Use the worksheets below and on the next page to identify 2 other classroom situations in which the ratio comparison of the number of girl students compared to the number of boy students is equivalent to the ratio in your classroom.

Classroom 1
Number of girl students ____ , Number of boy students ____

Make diagrams that show that the number of girl students compared by ratio to the number of boy students is equivalent to _____ .
(**Fill in the blank with the ratio comparison, reduced to lowest terms, of the number of girl students compared to the number of boy students in your classroom.)**

Write a number sentence that shows that the number of girl students compared by ratio to the number of boy students is equivalent to _____ .

Number Sentence

Classroom 2
Number of girl students ____ , Number of boy students ____

Make diagrams that show that the number of girl students compared by ratio to the number of boy students is equivalent to _____ .
(**Fill in the blank with the ratio comparison, reduced to lowest terms, of the number of girl students compared to the number of boy students in your classroom.)**

Write a number sentence that shows that the number of girl students compared by ratio to the number of boy students is equivalent to _____ .

Number Sentence

Lesson 11 Vocabulary

Write down the new words that you learned in this lesson. Use each of the new words in a sentence.

LESSON 12
MEETING NUMBER COMPARISONS

LESSON OVERVIEW: In this lesson, we will make subtraction and ratio comparisons of features of Meeting Number experiments.

Class Work: Complete the "Subtraction and Ratio Comparisons Table" below.

Make **subtraction comparisons** and **ratio comparisons** of the **number of pulse beats** in the following pairs of pulse beat groups:

- 5-PBG compared to a 3-PBG
- 3-PBG compared to a 5-PBG
- 8-PBG compared to a 12-PBG
- 12-PBG compared to an 8-PBG
- 2-PBG compared to a 3-PBG
- 3-PBG compared to a 2-PBG
- 11-PBG compared to a 9-PBG
- 9-PBG compared to an 11-PBG

SUBTRACTION AND RATIO COMPARISONS TABLE

PBGs Being Compared	Subtraction Comparison	Ratio Comparison	Reduced Ratio Comparison
5-PBG and 3-PBG			
3-PBG and 5-PBG			
8-PBG and 12-PBG			
12-PBG and 8-PBG			
2-PBG and 3-PBG			
3-PBG and 2-PBG			
11-PBG and 9-PBG			
9-PBG and 11-PBG			

Class Discussion: Use information from the "Subtraction and Ratio Comparisons" table on the previous page to discuss the ideas below.

Give examples of equivalent subtraction comparisons and say how they are equivalent.

Give examples of equivalent ratio comparisons and say how they are equivalent.

Give examples of opposite comparisons and say why they are opposite.

In Lesson 11, we began to study opposite subtraction comparisons. In the table on the previous page, did you find any ratio comparisons that you think are opposite? Why or why not?

COMPARING PULSE BEAT GROUP REPETITION NUMBERS

On the previous pages, we made subtraction and ratio comparisons of the number of pulse beats in different pairs of pulse beat groups.

We will now make subtraction and ratio comparisons of the repetition numbers for different pulse beat groups. We will work with:

A 3-Pulse Beat Group,

A 4-Pulse Beat Group, and

A 12-Pulse Beat Group, which is the
Meeting Number Pulse Beat Group for a 3-PBG and 4-PBG.

MEETING NUMBER REPETITIONS TABLE
3-PULSE BEAT GROUP, 4-PULSE BEAT GROUP

MN(3,4)-PBG* (12-PBG) Repetitions	3-PBG Repetitions	4-PBG Repetitions	Total Number of Pulse Beats
1	4	3	12
2	8	6	24
3	12	9	36
4	16	12	48
5	20	15	60

* **MN(3,4)-PBG** – Meeting Number Pulse Beat Group for a 3-PBG and a 4-PBG

Team Work:
Use information from the Meeting Number Repetitions Table on the previous page to fill in the Comparison Tables on the next two pages for the following:

Compare the repetition numbers for a 4-Pulse Beat Group and a 12-Pulse Beat Group.

Compare the repetition numbers for a 3-Pulse Beat Group and a 12-Pulse Beat Group.

Compare the repetition numbers for a 3-Pulse Beat Group and a 4-Pulse Beat Group.

COMPARISON TABLE FOR 4-PULSE BEAT GROUP REPETITIONS AND 12-PULSE BEAT GROUP REPETITIONS

MN(3,4)-PBG (12-PBG) Repetitions	Subtraction Comparison-4-PBG Repetitions compared to 12-PBG Repetitions	Ratio Comparison-4-PBG Repetitions compared to 12-PBG Repetitions	Ratio Comparison Reduced to Lowest Terms
1			
2			
3			
4			
5			

Are there any equivalent subtraction comparisons in this Comparisons Table? ____ Why do you think that this is so?

Are there any equivalent ratio comparisons in the Comparisons Table? ____ Why do you think that this is so?

COMPARISON TABLE FOR 3-PULSE BEAT GROUP REPETITIONS AND 12-PULSE BEAT GROUP REPETITIONS

MN(3,4)-PBG (12-PBG) Repetitions	Subtraction Comparison-3-PBG Repetitions compared to 12-PBG Repetitions	Ratio Comparison-3-PBG Repetitions compared to 12-PBG Repetitions	Ratio Comparison Reduced to Lowest Terms
1			
2			
3			
4			
5			

Are there any equivalent subtraction comparisons in this Comparisons Table? ____ Why do you think that this is so?

Are there any equivalent ratio comparisons in the Comparisons Table? ____ Why do you think that this is so?

COMPARISON TABLE FOR 3-PULSE BEAT GROUP REPETITIONS AND 4-PULSE BEAT GROUP REPETITIONS

MN(3,4)-PBG (12-PBG) Repetitions	Subtraction Comparison-3-PBG Repetitions compared to 4-PBG Repetitions	Ratio Comparison-3-PBG Repetitions compared to 4-PBG Repetitions	Ratio Comparison Reduced to Lowest Terms
1			
2			
3			
4			
5			

Are there any equivalent subtraction comparisons in this Comparisons Table? ____ Why do you think that this is so?

Are there any equivalent ratio comparisons in the Comparisons Table? ____ Why do you think that this is so?

Class Work: Teams report out.

Class Discussion: Discuss the following questions.

What patterns did you observe in the subtraction comparisons of the different repetition numbers that you compared?

What patterns did you observe in the ratio comparisons of the different repetition numbers that you compared?

Class Work: Fill in the Comparison Tables below and on the next page. Describe the strategies that you use to find the missing information.

COMPARISON TABLE FOR
2-PULSE BEAT GROUP REPETITION NUMBERS
AND 3-PULSE BEAT GROUP REPETITION NUMBERS

MN(2,3)-PBG (__-PBG) Repetitions	Subtraction Comparison-2-PBG Repetitions compared to 3-PBG Repetitions	Ratio Comparison-2-PBG Repetitions compared to 3-PBG Repetitions	Ratio Comparison Reduced to Lowest Terms
1	3 – 2 = 1	3/2	3/2
2	6 – __ = __	6/__	__/__
3	__ – __ = __	__/6	__/__
4	__ – 8 = __	12/8	__/__
5	__ – __ = 5	__/__	__/__

List the equivalent ratios that you found.

COMPARISON TABLE FOR 3-PULSE BEAT GROUP REPETITION NUMBERS AND 2-PULSE BEAT GROUP REPETITION NUMBERS

MN(3,2)-PBG (__-PBG) Repetitions	Subtraction Comparison-3-PBG Repetitions compared to 2-PBG Repetitions	Ratio Comparison-3-PBG Repetitions compared to 2-PBG Repetitions	Ratio Comparison Reduced to Lowest Terms
1	2 – 3 = –__	2/3	2/3
2	4 – __ = –2	__/6	__/__
3	__ – __ = __	__/9	__/__
4	__ – 12 = __	8/12	__/__
5	__ – __ = –5	__/__	__/__

List the equivalent ratios that you found.

How do these two Comparison Tables connect to each other?

Team Work: Fill in the Comparison Tables below and on the next page.

COMPARISON TABLE FOR
6-PULSE BEAT GROUP REPETITIONS
AND 8-PULSE BEAT GROUP REPETITIONS

MN(6,8)-PBG (__-PBG) Repetitions	Subtraction Comparison-6-PBG Repetitions compared to 8-PBG Repetitions	Ratio Comparison-6-PBG Repetitions compared to 8-PBG Repetitions	Ratio Comparison Reduced to Lowest Terms
1	4 – 3 = 1	__/__	__/__
2	8 – __ = 2	8/__	__/__
3	__ – __ = __	12/__	__/__
4	16 – __ = __	__/12	__/__
5	__ – __ = 5	__/__	__/__

List the equivalent ratios that you found.

COMPARISON TABLE FOR 8-PULSE BEAT GROUP REPETITIONS AND 6-PULSE BEAT GROUP REPETITIONS

MN(8,6)-PBG (__-PBG) Repetitions	Subtraction Comparison-8-PBG Repetitions compared to 6-PBG Repetitions	Ratio Comparison-8-PBG Repetitions compared to 6-PBG Repetitions	Ratio Comparison Reduced to Lowest Terms
1	3 – 4 = –__	3/4	__/__
2	6 – __ = __	6/__	__/__
3	9 – __ = __	__/__	__/__
4	__ – 12 = __	12/16	__/__
5	__ – __ = –5	__/__	__/__

List the equivalent ratios that you found.

How do both of the Comparisons Tables connect with each other?

List the opposite comparisons that you found.

Lesson 12 Vocabulary

Write down the new words that you learned in this lesson. Use each of the new words in a sentence.

LESSON 13
Graphing Subtraction and Ratio Comparisons

Lesson Overview: In this lesson, we will study graphs of data from the Repetition Number Comparison Tables completed in Lesson 12.

This will prepare you to make graphs of subtraction and ratio comparisons for some of the other combinations of pulse beat groups.

You will also practice **analyzing** graphs, in other words, using graphs to identify patterns in (and to get information about) different combinations of pulse beat groups.

USING GRAPHS TO REPRESENT EVENTS

Mathematicians use graphs as one way of representing and studying different events and ideas. Representing data in a graph can help to identify patterns that exist in something that is being studied.

The more patterns that we are able to recognize, the better that we are able to make predictions.

Class Discussion:
What are some types of information that we can get from making graphs of the meeting number activity?

Class Work: Study and discuss the comparison tables and graphs on the following pages. The tables and graphs are based upon information contained in the Meeting Number Repetitions Table below. (This table, which you completed in Lesson 12, contains information about the number of 3-PBG and 4-PBG repetitions that occur for meeting number pulse beat group repetitions.)

MEETING NUMBER REPETITIONS TABLE
3-PULSE BEAT GROUP AND 4-PULSE BEAT GROUP

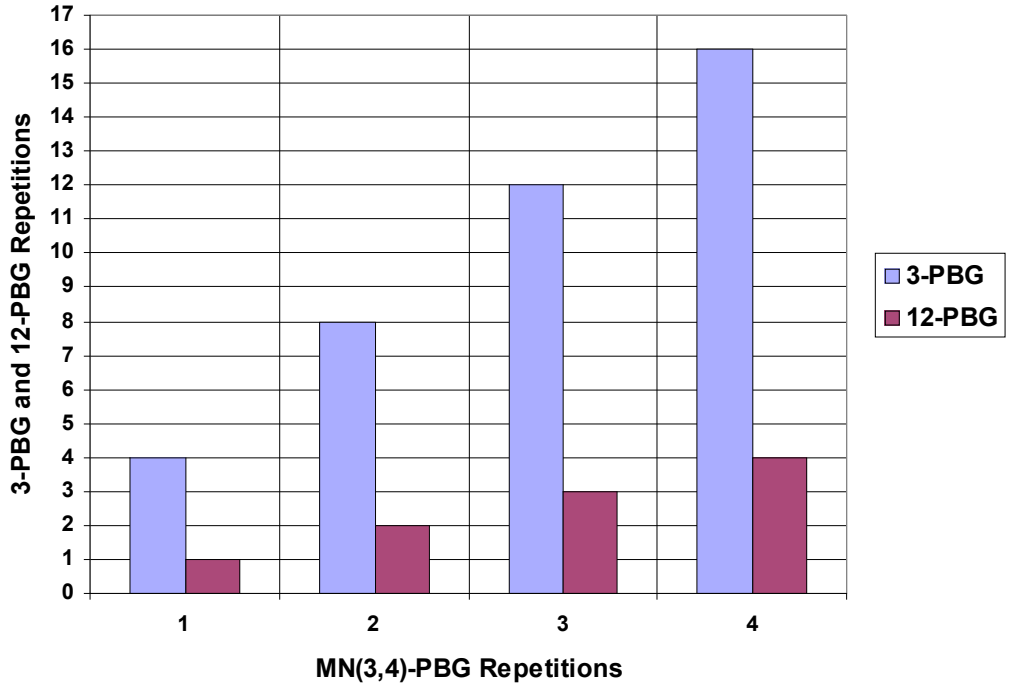
MN(3,4)-PBG* (12-PBG) Repetitions	3-PBG Repetitions	4-PBG Repetitions	Total Number of Pulse Beats
1	4	3	12
2	8	6	24
3	12	9	36
4	16	12	48
5	20	15	60

* **MN(3,4)-PBG** – Meeting Number Pulse Beat Group for a 3-PBG and a 4-PBG

COMPARISON TABLE FOR
3-PULSE BEAT GROUP REPETITIONS
AND 12-PULSE BEAT GROUP REPETITIONS

MN(3,4)-PBG (12-PBG) Repetitions	Subtraction Comparison-3-PBG Repetitions compared to 12-PBG Repetitions	Ratio Comparison-3-PBG Repetitions compared to 12-PBG Repetitions	Ratio Comparison Reduced to Lowest Terms
1	4 – 1 = 3	4/1	4/1
2	8 – 2 = 6	8/2	4/1
3	12 – 3 = 9	12/3	4/1
4	16 – 4 = 12	16/4	4/1

One type of graph that we can use to represent the information in this table is called a **bar graph**. Why do you think that the graph below is called a **bar graph**? What relationships do you see between the **bar graph** below and the comparison table above?



Class Discussion: Discuss the subtraction comparison and ratio comparison questions below. Use the comparison table and graph on the previous page to discuss these questions.

Subtraction Comparison Questions

For each Meeting Number Pulse Beat Group repetition, how much larger is the 3-PBG repetition number than the 12-PBG repetition number?

How is this information shown in the table?

How is this information shown in the bar graph?

Ratio Comparison Questions

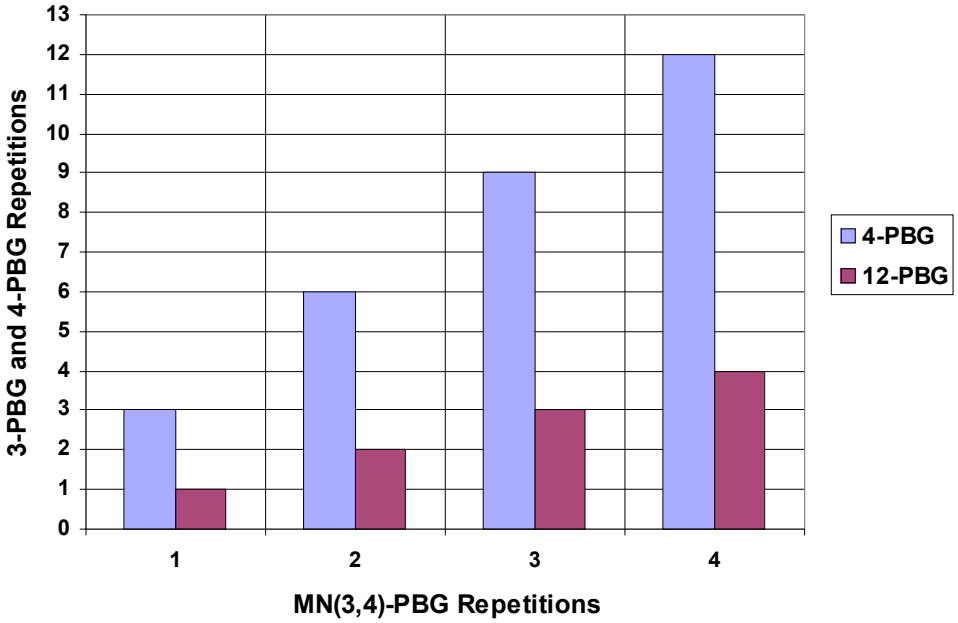
For each Meeting Number Pulse Beat Group repetition, how many times as large is the 3-PBG repetition number than the 12-PBG repetition number?

How is this information shown in the table?

How is this information shown in the bar graph?

COMPARISON TABLE FOR 4-PULSE BEAT GROUP REPETITIONS AND 12-PULSE BEAT GROUP REPETITIONS

MN(3,4)-PBG (12-PBG) Repetitions	Subtraction Comparison-4-PBG Repetitions compared to 12-PBG Repetitions	Ratio Comparison-4-PBG Repetitions compared to 12-PBG Repetitions	Ratio Comparison Reduced to Lowest Terms
1	3 – 1 = 2	3/1	3/1
2	6 – 2 = 4	6/2	3/1
3	9 – 3 = 6	9/3	3/1
4	12 – 4 = 8	12/4	3/1
5	15 – 5 = 10	15/5	3/1



Class Discussion: Discuss the subtraction comparison and ratio comparison questions below. Use the comparison table and graph on the previous page to discuss these questions.

Subtraction Comparison Questions

For each Meeting Number Pulse Beat Group repetition, how much larger is the 4-PBG repetition number than the 12-PBG repetition number?

How is this information shown in the table?

How is this information shown in the bar graph?

Ratio Comparison Questions

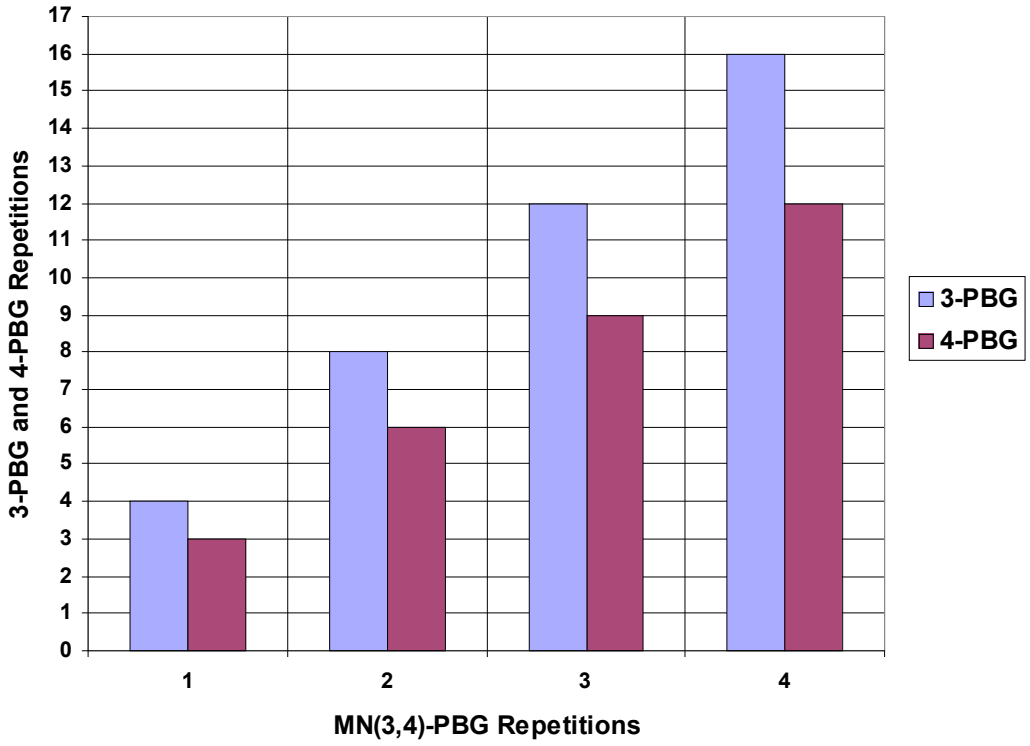
For each Meeting Number Pulse Beat Group repetition, how many times as large is the 4-PBG repetition number than the 12-PBG repetition number?

How is this information shown in the table?

How is this information shown in the bar graph?

COMPARISON TABLE FOR 3-PULSE BEAT GROUP REPETITIONS AND 4-PULSE BEAT GROUP REPETITIONS

MN(3,4)-PBG (12-PBG) Repetitions	Subtraction Comparison-3-PBG Repetitions compared to 4-PBG Repetitions	Ratio Comparison-3-PBG Repetitions compared to 4-PBG Repetitions	Ratio Comparison Reduced to Lowest Terms
1	4 – 3 = 1	4/3	4/3
2	8 – 6 = 2	8/6	4/3
3	12 – 9 = 3	12/9	4/3
4	16 – 12 = 4	16/12	4/3
5	20 – 15 = 5	20/15	4/3



Class Discussion: Discuss the subtraction comparison and ratio comparison questions below. Use the comparison table and graph on the previous page to discuss these questions.

Subtraction Comparison Questions

For each Meeting Number Pulse Beat Group repetition, how much larger is the 3-PBG repetition number than the 4-PBG repetition number?

How is this information shown in the table?

How is this information shown in the bar graph?

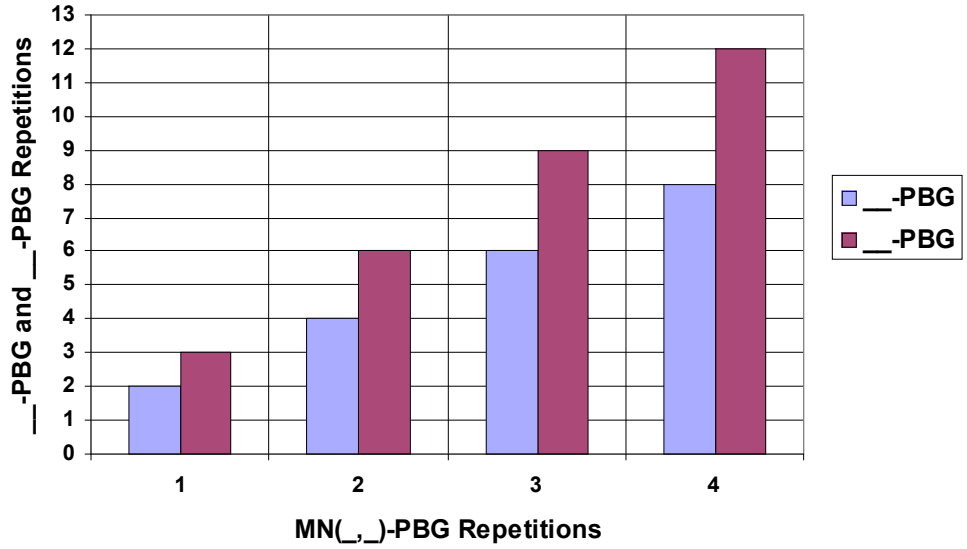
Ratio Comparison Questions

For each Meeting Number Pulse Beat Group repetition, how many times as large is the 3-PBG repetition number than the 4-PBG repetition number?

How is this information shown in the table?

How is this information shown in the bar graph?

Class Work: Study the bar graph below and discuss the questions that follow.

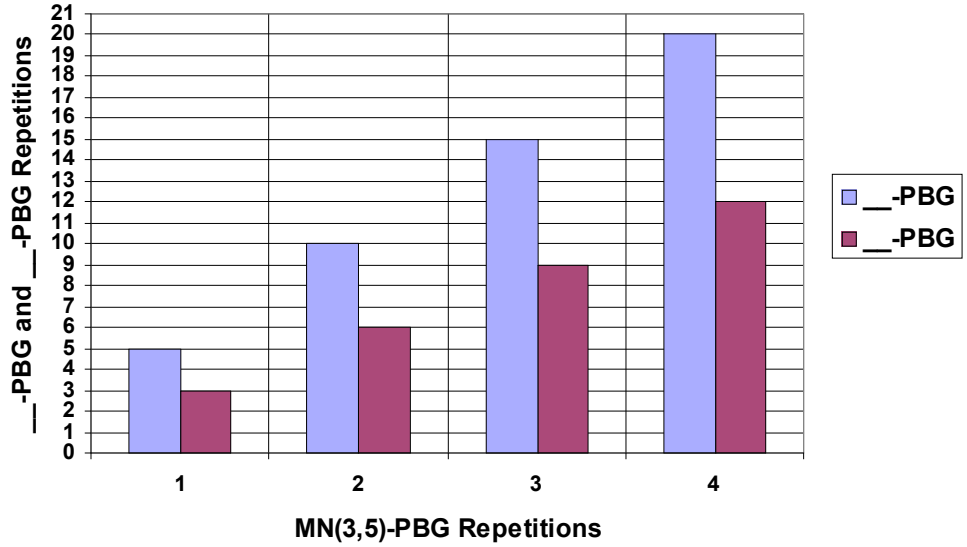


What different pairs of pulse beat groups could this bar graph represent?

What information would you need to have in order to identify for certain what pair of pulse beat groups is being compared?

Team Work: Complete the work that follows. Although, you will be working in teams, each student is responsible for all of the work and for being able to explain how you came up with your solutions.

Use the bar graph to answer the questions below.



Do you have enough information to figure out the sizes of the two different pulse beat groups that are being compared?

If yes, what are the two pulse beat groups?

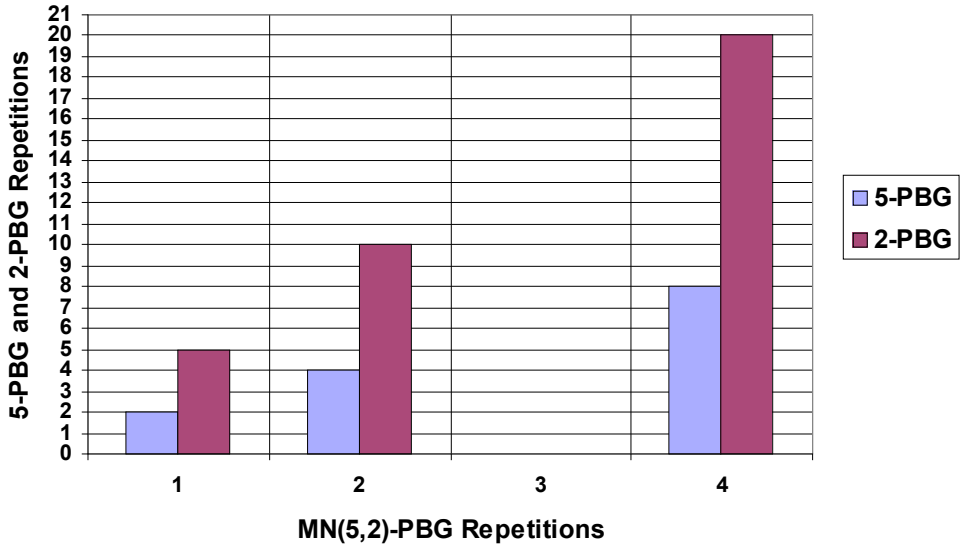
If no, what additional information do you need?

Do you have enough information to figure out the size of the meeting number pulse beat group?

If yes, what is the meeting number pulse beat group?

If no, what additional information do you need?

Team Work: Complete the bar graph below.

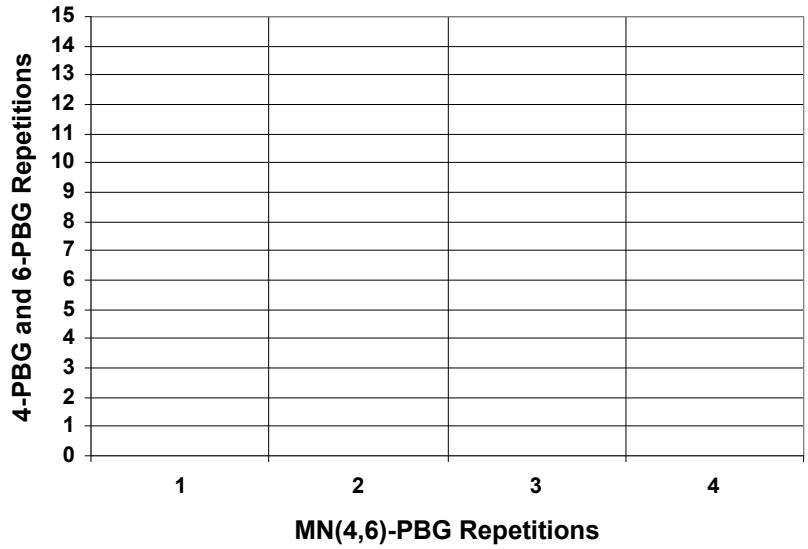


What strategies did you use to complete the graph?

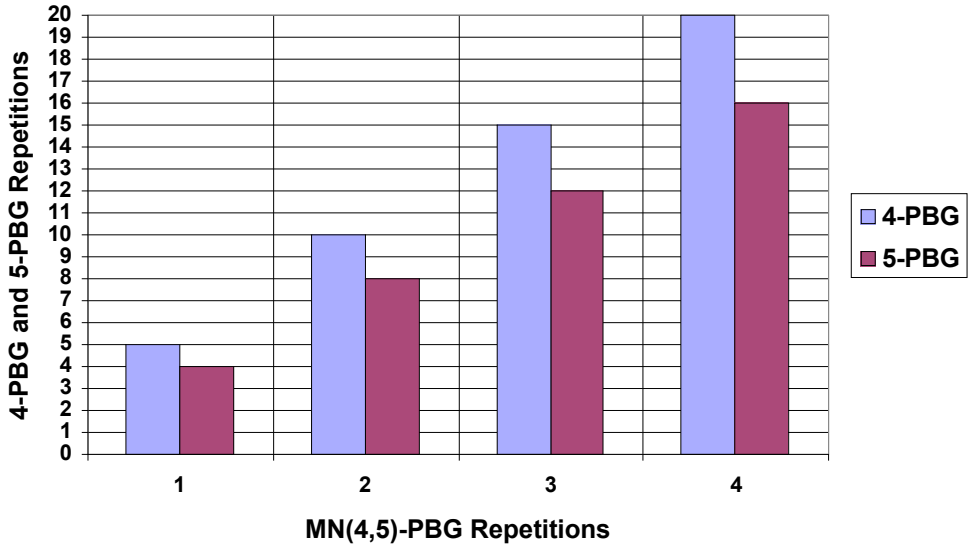
Team Work: Make a bar graph based upon the information contained in the Comparison Table below.

COMPARISON TABLE FOR
4-PULSE BEAT GROUP REPETITIONS
AND 6-PULSE BEAT GROUP REPETITIONS

MN(4,6)-PBG (__-PBG) Repetitions	Subtraction Comparison-4-PBG Repetitions compared to 6-PBG Repetitions	Ratio Comparison-4-PBG Repetitions compared to 6-PBG Repetitions	Ratio Comparison Reduced to Lowest Terms
1	3 – 2 = 1	3/2	3/2
2	6 – 4 = 2	6/4	3/2
3	9 – 6 = 3	9/6	3/2
4	12 – 8 = 4	12/8	3/2



Team Work: Use the bar graph to fill in the Comparison Table below.



COMPARISON TABLE FOR
__-PULSE BEAT GROUP REPETITIONS
AND __-PULSE BEAT GROUP REPETITIONS

MN(_,_)-PBG (__-PBG) Repetitions	Subtraction Comparison- __-PBG Repetitions compared to __-PBG Repetitions	Ratio Comparison- __-PBG Repetitions compared to __-PBG Repetitions	Ratio Comparison Reduced to Lowest Terms
1			
2			
3			
4			

Class Work: Teams report out and discuss the strategies used to complete the work.

Lesson 13 Vocabulary

Write down the new words that you learned in this lesson. Use each of the new words in a sentence.

Message to Students, Teachers & Drummers:

Congratulations! You have completed the Algebra Project's
African Drums & Ratios Curriculum
Pulse & Meeting Number module ☺

Thank you for participating in this journey through
African Diasporic Drumming and Mathematics!

Optional Class Activity: The Class may want to talk about how to organize a presentation about the African Drums & Ratios Curriculum to the rest of the school community. You may use the space below to take notes on the class discussion.

LESSON A
DESCRIBING
THE MEETING NUMBER ACTIVITY

Individual Work: Write a **five-paragraph description** of the Meeting Number experiments. The questions below will guide and help you include all the information that you need for each of the paragraphs that you write. Use complete sentences in your writing.

As you answer these questions, make the information clear and detailed enough so that students in another class would be able to do the experiments based upon what you write--even if they have never seen the meeting number experiments in person. You may also include drawings to help make your descriptions clearer. Your teacher will provide you with paper to write (and draw) your descriptions.

Paragraph One
What questions were the Meeting Number experiments set up to answer? What supplies do you need to do the Meeting Number experiments?

Paragraph Two
What are the different parts that were used in the Meeting Number experiments?

Paragraph Three
How were the experiments done?

Paragraph Four
What were the results of the experiments? What surprised you? What things did you learn? (Identify three or more things)

Paragraph Five
What other Meeting Number experiment do you think would be interesting to try? (What would you be trying to figure out in doing this experiment?)

Team Work: Share your descriptions of the activity with each other. Make a list of the main **items, parts, or features** included in your descriptions.

Main Items, Parts, or Features of the Meeting Number Activity Team List

Put your list on chart paper and choose a team member (or members) to report out to the whole class.

Class Work: Teams report out their charts from the previous page. As teams report, create a class list, below.

Main Items, Parts, or Features of the Meeting Number Activity Class List

How we Develop Mathematics in the Algebra Project: A Five-Step Learning Process

Your teacher may have described the five-step process used in the Algebra Project to help students learn mathematics. The five steps are:

1. Students experience a physical event.

2. Students draw pictures of the event.

3. Students write and talk about the event in their everyday language (**ordinary talk**). This is the talk we use in our everyday languages about the people we know, the objects we use and the events we experience and observe.

4. Students discuss the event using the type of formal language used by mathematicians and scientists to structure our everyday talk (**feature talk**). This type of talk places a focus on key features and relationships that are involved in the physical event.

5. Students develop and use symbols to represent features of the event and various ways that these features can be related to each other (**symbolic representation**).

NOTE: Robert P. Moses, Founder and President of the Algebra Project Inc., developed this five-step learning process, based upon his own insights about how "grassroots" people became empowered in small, local Civil Rights meetings in Mississippi in the early 1960s, and through his studies of researchers of education, of mathematics and of philosopy. Among these researchers are John Dewey, Jean Piaget, William Van Orman Quine, Kurt Lewin, and David A. Kolb.

Class Discussion: Give examples of things that we have done in the lessons so far and what steps from the Algebra Project 5-Step Learning Process you think that these examples fit with?

Meeting Number Experiments

Physical Events	Drawing Pictures	Ordinary Talk	Feature Talk	Symbolic Representation

Identifying Features of the Meeting Number Activity

As you have described in your work, the meeting number activity has many **features**. Paying close attention to different **features** of an event can lead to different kinds of learning. In the African Drums & Ratios Curriculum we use the following features of the meeting number activity to develop some important mathematics ideas:

- pulse
- pulse beat
- pulse beat groups
- pulse beat count number
- the number of pulse beats per group (number of pulse beats in a pulse beat group, or the size of a pulse beat group)
- number of pulse beat groups
- meeting number
- the total number of pulse beats

Lesson A Vocabulary

Write down the new words that you learned in this lesson. Use each of the new words in a sentence.

LESSON B
MAKING SYMBOLS

Class Discussion: What are symbols? How are they used? In the table below, in the "Symbol" column, are examples of various symbols you may be familiar with. In the "Meaning" column, write what these symbols mean to you.

Symbol	Meaning
WWW.	
÷	
%	
@	

Where do symbols that you use come from?

How do symbols get their meanings?

Team Work: Think of other symbols that you have seen before and write or draw them in the "Symbol" column in the table below. In the "Meaning" column, write what these symbols mean to you.

Other Symbols That You've Seen Before

Symbol	Meaning

Copy this table, along with the symbols, onto chart paper. Leave the "Meanings" column blank. When your team reports out, your classmates will attempt to find the meanings.

Think of some ideas that you want to express or some objects that you want to represent. Make up your own symbols for these ideas or objects and draw/write them in the "Symbol" column in the table below. In the "Meaning" column write what these symbols mean to you.

Symbols that You Created

Symbol	Meaning

Copy this table, along with the symbols that you created, onto chart paper. This time, leave the "Meaning" column blank. When your team reports out, the rest of the class will attempt to guess your symbols' meanings.

Class Work: Teams report out. As teams report out, the rest of the class attempts to determine the meanings of the symbols chosen and created by each team.

Class Discussion: Discuss the questions below.

Was it easier to understand any of the symbols? Why or why not?

Do you think that these symbols mean the same thing to everyone in the world? Why or why not?

Give examples of other kinds of symbols, such as "movement symbols" (gestures, etc.) and "sound symbols" (such as the NBC television identifying sound, or a school bell). As you identify and demonstrate (if possible) examples, list them in the table below.

Movement Symbols	**Sound Symbols**

MAKING SYMBOL SENTENCES

Team Instructions: You will work in teams to develop **symbol sentences**. Because words are one type of symbol used to represent people, objects, actions, emotions and ideas, a word sentence is one type of **symbol sentence**.

In your teams, you will develop symbol sentences that don't use words. This means that you will need to use a string or sequence of symbols to express messages. You will use visual (drawn) symbols, sound symbols, and movement symbols.

Team Work: Create symbol sentences for two messages. One message has been provided for you. You will need to decide upon the second one yourselves. You will create visual symbols, sound symbols, and movement symbols for each message.

Word Sentence
Together, we can make a better world.

Visual Symbol Sentence
In the box below, develop a sequence of drawn symbols to express this idea.

Sound Symbol Sentence
Develop a sequence of sound symbols to express this idea.

Movement Symbol Sentence
Develop a sequence of movement symbols to express this idea.

Word Sentence
Decide upon a word message and write it on the line below.

_____

Visual Symbol Sentence
In the box below, develop a sequence of drawn symbols to express this idea.

Sound Symbol Sentence
Develop a sequence of sound symbols to express this idea.

Movement Symbol Sentence
Develop a sequence of movement symbols to express this idea.

Class Work: Teams report out.

Lesson B Vocabulary

Write down the new words that you learned in this lesson. Use each of the new words in a sentence.

LESSON C
MAKING MATHEMATICS SYMBOLS

Lesson Overview: In this lesson, you will develop symbol sentences to express mathematical ideas from the meeting number activities with which you have been working.

Class Discussion: One important part of studying mathematics is learning to recognize and use the symbols that express mathematics ideas. Over the centuries, people from many cultures developed and used their own mathematics symbols.

As time has passed, more and more symbols have been accepted into widespread usage, so that mathematicians in one part of the world can communicate with mathematicians in another part of the world using mathematical symbols. Below are examples of symbols that have been used by people from many countries and different cultures to represent numbers. Some of these symbols are still in use and some are not.

NUMBER SYMBOLS FROM AROUND THE WORLD*

Hindu Number Symbols	Egyptian Number Symbols
Chinese Number Symbols	Hebrew Number Symbols

* Smith, David E., (1919). Number Stories of Long Ago. NCTM, Washington, DC.

MORE NUMBER SYMBOLS FROM AROUND THE WORLD

Babylonian Number Symbols	Roman Number Symbols
European Number Symbols	Greek Number Symbols

How do you think that these number symbols from around the world came into use?

Why do you think that some symbols spread around the world and some remained only in one area?

MEETING NUMBER ACTIVITIES SUMMARY

Thus far, you have experimented with playing different combinations of Pulse Beat Groups along with a Pulse. You have developed and used different strategies for:

- predicting Meeting Numbers;
- finding the size of Meeting Number Pulse Beat Groups, and;
- finding Pulse Beat Group Factors.

You have also explored some of the different relationships that exist among different size Pulse Beat Groups, their Meeting Number Pulse Beat Groups, and Pulse Beat Group Factors.

MEETING NUMBER SYMBOL SENTENCES

Class Discussion: Use symbols to make "Symbol Sentences" that describe some of the various characteristics of the Meeting Number Activity.

To construct these symbol sentences, you will need to use what we call "feature talk" in the Algebra Project's African Drums & Ratios Curriculum. Feature talk is a little bit different from the "ordinary talk" that we usually use to describe things that we experience.

For example, in "ordinary talk", we might say

"A 4-Pulse Beat Group

is Larger than

a 3-Pulse Beat Group."

In a "feature talk" sentence, emphasis is put on the feature that is being compared. For example, in feature talk, we would say

"The number of pulse beats in a 4-Pulse Beat Group

is Greater Than

the number of pulse beats in a 3-Pulse Beat Group."

What is the feature being compared in the example above?

_____

Team Work: Make visual (drawing/picture) symbol sentences and number sentences for the nine feature talk sentences on the following pages.

After you have completed these symbol sentences in your workbooks, copy them onto chart paper in preparation for reporting out to the whole class.

First Feature Talk Sentence:
The number of pulse beats in a 4-Pulse Beat Group is greater than the number of pulse beats in a 3-Pulse Beat Group.

Visual Symbol Sentence

Number Sentence

$4 > 3$

Second Feature Talk Sentence:
The number of pulse beats in a 3-Pulse Beat Group is less than the number of pulse beats in a 4-Pulse Beat Group.

Visual Symbol Sentence

Number Sentence

$3 < 4$

Third Feature Talk Sentence:
The number of pulse beats in a 3-Pulse Beat Group is the same as the number of pulse beats in a 3-Pulse Beat Group.

Visual Symbol Sentence

Number Sentence

3 = 3

Fourth Feature Talk Sentence:
The number of pulse beats in a 1000-Pulse Beat Group is greater than the number of pulse beats in a 925-Pulse Beat Group.

Visual Symbol Sentence

Number Sentence

Fifth Feature Talk Sentence:
The number of pulse beats in a 925-Pulse Beat Group is less than the number of pulse beats in a 1000-Pulse Beat Group).

Visual Symbol Sentence

Number Sentence

Sixth Feature Talk Sentence:
The number of pulse beats in a 1000-Pulse Beat Group is the same as the number of pulse beats in a 1000-Pulse Beat Group).

Visual Symbol Sentence

Number Sentence

Team Work: Choose your own pulse beat groups to use in the comparisons that follow. In addition to "Feature Talk Sentences", "Visual Symbol Sentences", and "Number Sentences", also develop Sound Symbol Sentences using drums and/or other percussion instruments.

Seventh Feature Talk Sentence:
The number of pulse beats in a ___-Pulse Beat Group is greater than the number of pulse beats in a ___-Pulse Beat Group).

Visual Symbol Sentence

Number Sentence

Sound Symbol Sentence
Develop a sound symbol sentence using drums and/or other percussion instruments.

Eighth Feature Talk Sentence:
The number of pulse beats in a ___-Pulse Beat Group is less than the number of pulse beats in a ___-Pulse Beat Group).

Visual Symbol Sentence

Number Sentence

Sound Symbol Sentence
Develop a sound symbol sentence using drums and/or other percussion instruments.

Ninth Feature Talk Sentence:
The number of pulse beats in a ___-Pulse Beat Group is the same as the number of pulse beats in a ___-Pulse Beat Group).

Visual Symbol Sentence

Number Sentence

Sound Symbol Sentence
Develop a sound symbol sentence using drums and/or other percussion instruments.

Class Work: Teams report out. Use the space below to take notes.

Team Work: The symbol sentences that you created on the previous pages involved the use of combinations of individual symbols. Fill in the tables below with the different symbols that you used in your symbol sentences.

Feature	**Symbol**
Pulse Beat	
Number of Pulse Beats	
3-Pulse Beat Group	
4-Pulse Beat Group	
925-Pulse Beat Group	
1000-Pulse Beat Group	
___-Pulse Beat Group	
Meeting Number-Pulse Beat Group	

Comparison	Symbol
Is Greater Than	
Is Less Than	
Is the Same As	

Class Discussion: Discuss what you and your classmates learned about symbols. In the space below, number and write down **5 or more** ideas you and your classmates shared in the discussion:

IDEAS WE LEARNED ABOUT SYMBOLS:

Lesson C Vocabulary

Write down the new words that you learned in this lesson. Use each of the new words in a sentence.