

FIVE
MISCONCEPTIONS
ABOUT
CALCULUS
ACCESS



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THE FIVE MISCONCEPTIONS

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5 Misconceptions About Calculus Access

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By William Crombie | The Algebra Project, Inc.

Introduction: The Prerequisites Myth That's Failing Our Students

Do you want to increase access to advanced mathematics in your school district? Peeling away the five misconceptions about calculus access will enable your students and teachers to do just that!

Every year, mathematics departments across the United States systematically exclude hundreds of thousands of students from calculus study. Not because these students lack mathematical capacity. Not because they can't handle sophisticated reasoning. But because we've constructed an elaborate mythology around calculus prerequisites that serves institutional gatekeeping more than mathematical understanding.

The numbers reveal the crisis. According to the 2024 National Assessment of Educational Progress (NAEP), only 22% of 12th graders perform at or above proficient levels in mathematics, while 45% score below Basic level. The lack of math literacy falls hardest on Black and Brown communities, though all communities suffer: only 5% of Black 12th graders, 8% of Native American, 9% of Latino/a, 25% of two or more races, 28% of white, and 53% of Asian and Pacific Islander 12th graders scored proficient in 2024. Those 95% of Black 12th graders not proficient represent more than 490,000 students in a single year, while the 68% of white 12th graders not proficient represent more than 1,200,000 students. These disparities don't represent differences in mathematical capability—they represent the cumulative effects of a system that filters out rather than develops mathematical talent.

Here's what should concern every district supervisor: much of this exclusion is unnecessary. Twenty years of research and classroom implementation by the Algebra Project demonstrate that core calculus concepts are accessible to students with basic algebra and geometry backgrounds. The barriers we've erected are largely artificial, created by historical accident and institutional choice rather than mathematical necessity.

When I developed a geometric quadrature of the parabola in 2000, determining areas under curved boundaries using only high school geometry, it became clear we had been asking the wrong question. Instead of "What prerequisites do students need for calculus?" we should ask "What artificial barriers prevent students from accessing mathematical concepts they could already understand?"

This report examines five persistent misconceptions about calculus prerequisites currently limiting your district's ability to prepare students for STEM careers and college success. These aren't merely pedagogical errors—after twenty years developing accessible mathematics approaches, I've come to recognize them as systemic barriers we've mistaken for mathematical truths.

The evidence comes from two decades of Algebra Project research and classroom implementation, including the past two years through our Accessible Calculus Project, supported by an initial grant from the National Science Foundation, currently demonstrating these principles in high schools in New Jersey and Georgia. The stakes couldn't be higher. Mathematics serves as the primary gatekeeper to STEM careers, but our current approach systematically excludes the diverse talent our economy desperately needs. As our late founder Bob Moses observed, we're creating "sharecropper education" where communities are denied mathematical literacy that then becomes the condition for blocking their greater economic and civic participation.

Districts that confront these misconceptions honestly will discover something remarkable: most students possess the mathematical reasoning capacity necessary for calculus concepts—we've just been presenting those concepts in ways that create artificial barriers. The real question isn't whether students can learn calculus. The question is whether we as educators have the courage to admit that our traditional prerequisite structures serve institutional convenience more than mathematical understanding.

Misconception #1: "Students Need Four Years of High School Mathematics to Understand Calculus"

The Problem

Walk into any high school mathematics department and you'll encounter an unquestioned orthodoxy: students must complete Algebra I, then Geometry, then Algebra II, then Pre-Calculus before they can possibly understand calculus concepts. This four-year sequential lockstep has become so thoroughly normalized that we rarely examine whether it reflects mathematical necessity or institutional intransigence.

The traditional sequence functions less as a developmental pathway than as a filtration system. Each course acts as a checkpoint where students are quietly excluded. A stumble in Algebra I often permanently forecloses access to higher mathematics. The system operates through cumulative disadvantage—early setbacks cascade into permanent mathematical marginalization.

But here's where our current systems fail most profoundly: we teach quadratic functions in Algebra I while systematically concealing their most important mathematical relationship—their connection to rates of change forming the conceptual foundation of the differential calculus. This isn't accidental pedagogy; it's *structural concealment*. We possess the mathematical tools to make derivative concepts accessible within Algebra I itself, yet we choose not to deploy them.

The Reality

Consider a revealing diagnostic exercise we conduct with high school mathematics teachers. When asked about the geometric meaning of the parameters in a linear function, expressed as $y = mx + b$, teachers respond confidently: m is the slope of the line, b is its y -intercept. But when asked about the expression for a quadratic function describing a parabola, $y = ax^2 + bx + c$, responses become hesitant, specifically around the linear parameter, b . The parameter a controls the opening and direction of the parabola, b somehow involves the axis of symmetry or a shift in the parabola, and c is the y -intercept. This uncertainty around the linear coefficient, b , isn't a minor pedagogical gap—it represents a systematic blind spot and the elimination of calculus concepts mathematically accessible within Algebra I itself.

Through twenty years of using Accessible Calculus concepts with teachers and their students, we've demonstrated that students can understand the complete geometric meaning of quadratic coefficients using reasoning they already possess. When students work systematically through polynomial construction—building from constant to linear to quadratic terms—a remarkable pattern of structural emergence becomes visible:

For $y = c$: The constant describes the height of the entire graph—a global property characterizing every point.

For $y = bx + c$: Adding the linear term transforms the constant's global property (height) into a local property at the y -intercept, while the linear coefficient b now describes the direction of the entire graph—a new global property.

For $y = ax^2 + bx + c$: Adding the quadratic term transforms the linear coefficient's global property (direction) into a local property at the y -intercept. The coefficient a describes how the graph is turning (its curvature), and the linear coefficient b now represents the direction of the graph at the y -intercept—the slope of the tangent line, which is precisely the conceptual foundation of derivatives.

What goes unrecognized in standard curriculum is this fundamental property of polynomials: each higher-order term describes how quickly the previous term's coefficient changes. Polynomial coefficients are rates of rates of change—a cascade of derivatives encoded in the algebraic structure itself.

Students working with the expression $y = 2x^2 - 3x + 2$ can immediately read the direction of the graph at the y -intercept: a slope of -3. This is genuine derivative thinking, accessible through algebraic and geometric reasoning students already command (Crombie & Grant, 2012). This approach doesn't sacrifice mathematical rigor—it enhances conceptual understanding by revealing connections traditional curriculum sequencing omits or obscures.

This exemplifies a broader principle: rigorous mathematical understanding develops through connection rather than compartmentalization. When students see relationships between algebraic, geometric, and calculus concepts—when

they recognize how these domains illuminate each other—they develop more robust and flexible mathematical knowledge (Hiebert & Carpenter, 1992). The traditional sequence, with its artificial boundaries between courses, actively works against students and teachers making this connective understanding.

What This Means for Your District

The systematic concealment of calculus-algebra connections creates artificial separations that perpetuate gatekeeping functions. Students manipulate quadratic functions for two years, in Algebra I and Algebra II, without understanding their most essential mathematical property. This isn't a curricular accident—it's a structural design that maintains calculus as a scarce credential.

Immediate Curricular Opportunities: Algebra I teachers can reveal the complete geometric meaning of quadratic coefficients as a natural extension of existing instruction—no additional time required. Students can encounter derivative concepts through familiar algebraic contexts, building conceptual bridges rather than disciplinary walls.

Equity Implications: The prerequisite structure creates systematically different experiences for different populations. Students from well-resourced schools often encounter these algebra-calculus connections through informal teacher knowledge or summer enrichment. Students from under-resourced schools experience calculus as completely disconnected from algebra, creating psychological barriers that persist throughout their development. The four-year requirement functions as educational hierarchy reproduction, concentrating advanced mathematical access among already-advantaged populations.

The Political Economy of Prerequisites: We should be explicit about what the prevailing four-year prerequisite accomplishes: it maintains calculus as a scarce resource accessible primarily to students who navigate four years of sequential mathematics without significant disruption—privileging students from stable, well-resourced backgrounds. It provides intellectual justification for tracking systems sorting students by perceived ability (which correlates tightly with race and class). When we challenge the four-year prerequisite, we're arguing students should learn mathematics more honestly—with connections visible rather than concealed, with advanced concepts accessible rather than artificially delayed. We're also not proposing students learn less rigorous mathematics. We're arguing that rigor should be built on understanding rather than replacing it. Twenty years of work with the Algebra Project and the accessible calculus demonstrates that students can engage in sophisticated mathematical reasoning through concrete foundations.

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Misconception #2: "Limits Are Essential for Understanding Calculus"

The Problem

Perhaps no mathematical concept has erected more unnecessary barriers to calculus access than the traditional insistence on limits as prerequisite knowledge. The conventional high school mathematics sequence forces students to master approximations, then refinements as infinite processes, and finally the limit formalism before they encounter any reason to care about calculus—before they see how these advanced techniques address questions about motion, growth, optimization, and change that make the subject intellectually compelling.

This entrenched pedagogical architecture inverts the natural order of mathematical understanding. It leads with the most technically demanding abstractions rather than building from concrete problems and geometric intuitions. Students are asked to memorize limit procedures without genuine comprehension while remaining mystified about what calculus actually does. The formal machinery obscures rather than illuminates the mathematical relationships which give calculus its power.

Students who struggle with limit formalism are automatically excluded from calculus study, even when they might excel at geometric reasoning, pattern recognition, and conceptual synthesis—the genuine foundations of calculus thinking. Let's face it: we've constructed an artificial barrier reflecting particular choices about curricular sequencing, not mathematical necessity.

The Reality

Historical analysis exposes the contingency of this limit-first approach. Newton and Leibniz developed calculus without the modern concept of limits. They understood derivatives as rates of change and integrals as accumulated area through algebraic and geometric reasoning—reasoning any student with foundational literacy in algebra and geometry can follow (Boyer, 1959; Edwards, 1979). The limit formalization came later, as 19th-century mathematicians sought rigorous foundations for results Newton and Leibniz had already discovered through more intuitive methods.

The Accessible Calculus Project demonstrates this principle systematically. Students can develop and understand the argument that the area under the parabola, $y = x^2$, from 0 to 1 is equal to $1/3$ the area of the enclosing square through a geometric construction using only transformations from Common Core geometry standards (Crombie & Grant, 2012). This becomes the gateway to the integral calculus—accessible through elementary means, yet conceptually sophisticated.

Students grasp derivatives as instantaneous rates of change through examining patterns in polynomial behavior. The fundamental theorem of calculus becomes transparent when approached through the analysis of boundaries implicit in Cavalieri's Principle—a principle which students understand from secondary geometry (Stein, 1999). These aren't simplified versions of calculus; they're genuine calculus reasoning reconstructed through elementary foundations.

This approach exemplifies an alternative structural emergence: complex mathematical ideas arising naturally from systematic attention to elementary relationships. When students encounter calculus through this lens, they develop conceptual understanding making formal limit theory more meaningful later. They're better prepared for abstraction because the abstractions now formalize relationships they already understand concretely.

What This Means for Your District

Requiring limits as prerequisite for the study of calculus artificially restricts your calculus pipeline to students who can tolerate high abstraction without motivating context—a cognitive style correlating with socio-economic privilege in ways that should make us deeply uncomfortable.

Curricular Architecture: Students can encounter genuine calculus concepts through geometric and algebraic reasoning accessible in Algebra I and Geometry. Elementary calculus approaches can serve as conceptual foundation for later formal development with limits. Multiple entry pathways accommodate diverse cognitive strengths rather than privileging abstract formalism.

The Political Economy of Prerequisites: The limits-first requirement maintains calculus as a scarce credential accessible primarily to students from well-resourced schools with extensive AP/IB preparation programs. It preserves status hierarchies within mathematics education. It provides intellectual cover for tracking systems reproducing existing patterns of racial and economic stratification. The question isn't whether limits matter—they obviously do for rigorous foundations. The question is whether the advanced techniques of finding limits must serve as gatekeepers to calculus understanding. Twenty years of work with the accessible calculus suggests they need not.

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Misconception #3: "Only 'Mathematically Gifted' Students Can Handle Calculus"

The Problem

The mythology of *mathematical giftedness* serves as perhaps the most insidious barrier to calculus access. When we reserve calculus for students labeled "mathematically gifted," we're not identifying innate ability—we're ratifying the results of a sorting system that systematically advantages some populations while excluding others.

The "giftedness" label typically gets applied based on performance in traditional mathematics sequences that already embed class and racial bias. Students who struggle with procedural fluency or timed testing but possess strong mathematical reasoning get excluded. Students whose schools lack resources for enrichment programs get excluded. Students whose life circumstances create barriers to consistent school participation get excluded. The label doesn't identify mathematical potential—it codifies accumulated educational advantage.

More fundamentally, the giftedness framework misunderstands the nature of mathematical ability. Mathematical reasoning isn't a fixed trait distributed unequally across populations. It's a capacity that develops through opportunity, instruction, and engagement with meaningful mathematical problems. When we restrict calculus to supposedly gifted students, we're not honoring mathematical talent—we're creating a self-fulfilling prophecy that ensures only certain students develop the mathematical identities and skills that calculus coursework provides.

The Reality

Research in mathematical cognition consistently demonstrates that mathematical capability is far more plastic and widespread than traditional gatekeeping structures acknowledge. Carol Dweck's work on growth mindset shows that students' beliefs about mathematical ability matter more than supposedly innate traits (Dweck, 2006). Jo Boaler's research demonstrates that heterogeneous grouping and accessible instruction produce better mathematical outcomes across all achievement levels than tracking and ability grouping (Boaler, 2008, 2016).

The Accessible Calculus Project provides concrete demonstration of this principle. When calculus concepts are presented through elementary algebraic and geometric foundations, students previously excluded from advanced mathematics demonstrate sophisticated mathematical reasoning. They discover derivative relationships in polynomial structure. They construct geometric arguments for integral calculations. They develop mathematical insights that traditional instruction reserves for supposedly gifted students.

What changes isn't student ability—it's instructional design. When we remove artificial barriers, when we build from familiar foundations, when we make mathematical relationships transparent rather than mysterious, more students demonstrate the mathematical capacity that was always present but previously untapped.

International comparisons support this finding. Countries with less tracking and broader access to advanced mathematics—countries that don't sort students into gifted and non-gifted categories as rigidly as the U.S.—produce higher mathematical achievement across wider populations (Schmidt et al., 2005). The giftedness framework doesn't identify mathematical talent; it suppresses it.

What This Means for Your District

The giftedness paradigm for calculus access creates artificial scarcity that serves institutional habit rather than student development. It protects status hierarchies. It provides a flawed intellectual justification for tracking systems that concentrate opportunity among already-advantaged populations.

Rethinking Placement: School districts should examine how mathematics course placement decisions are made. Are we evaluating mathematical reasoning and problem-solving capacity, or just procedural fluency developed through extensive preparation? Are we measuring mathematical potential, or accumulated educational advantage? Placement systems that rely primarily on standardized test scores and prerequisite course grades systematically reproduce existing inequalities.

Equity Implications: The giftedness framework falls hardest on students from communities historically excluded from advanced mathematics. When calculus becomes a marker of giftedness rather than fundamental literacy, when access gets restricted to supposedly elite students, we guarantee that existing patterns of educational inequality persist. Expanding calculus access isn't about lowering standards—it's about recognizing that current standards measure accumulated advantage rather than mathematical capability.

Mathematical Identity Development: Students develop mathematical identities through participation in mathematical learning communities, through effort and engagement with sophisticated mathematical ideas. When we reserve calculus for supposedly gifted students, we deny other students the experiences that would develop their mathematical identities and capabilities. This creates a self-fulfilling prophecy: students excluded from calculus never develop the mathematical confidence and skills that calculus participation would provide (Martin, 2009).

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Misconception #4: "Calculus Requires Abstract Mathematical Thinking"

The Problem

Traditional calculus instruction privileges abstract symbolic manipulation over concrete problem-solving and conceptual understanding. Students learn to compute derivatives and integrals through procedural algorithms—memorizing rules for power functions, products, quotients, chains—without understanding why these operations matter or how they connect to the phenomena that originally motivated calculus development. The subject appears as a collection of techniques for manipulating symbols rather than a powerful framework for reasoning about change.

This abstract emphasis creates a set of overlapping barriers. Students who excel at concrete reasoning, geometric visualization, and applied problem-solving but struggle with decontextualized symbolic manipulation get excluded from calculus study—despite possessing precisely the mathematical thinking skills that calculus applications actually require. Meanwhile, students who can execute symbolic procedures often lack the conceptual understanding necessary for applying calculus to novel problems.

The pedagogical inversion here is profound. We teach calculus as though abstraction were the foundation when historically and cognitively, abstraction is the outcome. Newton and Leibniz developed calculus through concrete engagement with problems of motion, tangent lines, and area calculation. The abstract formalism came later. Yet we reverse this developmental sequence, leading with abstractions that have no motivating context.

But we should also ask the political question: whose interests does this abstract emphasis serve? The privileging of abstract manipulation over concrete reasoning functions as a selection mechanism favoring students with particular cognitive styles and extensive preparation in decontextualized symbol manipulation—characteristics correlating predictably with race and class privilege. The abstraction requirement maintains calculus as an elite credential rather than as a fundamental literacy.

The Reality

Calculus emerges from concrete questions about motion, growth, optimization, and change—questions that can be approached through geometric visualization and algebraic reasoning accessible to any student with foundational literacy in high school mathematics. The abstract formalism, while mathematically elegant and historically important, often obscures rather than illuminates the conceptual relationships that give calculus its power.

The Accessible Calculus Project demonstrates systematically how calculus concepts can be presented through concrete foundations while maintaining mathematical rigor. Students understand derivatives through examining geometric slopes and discovering algebraic patterns in polynomial behavior. They encounter integrals through area calculation using mean value rectangles and area-preserving transformations requiring only geometric reasoning from high school standards. The fundamental theorem of calculus becomes transparent rather than mysterious when approached through an analysis implicit in Cavalieri's Principle. (Crombie, 2024)

Consider the derivative as an exemplar. Traditional instruction defines derivatives through the limit formalism: $f'(x) = \lim_{h \rightarrow 0} [f(x+h) - f(x)]/h$. This abstract definition mystifies students, who must manipulate symbols without clear geometric or physical intuition. But derivatives have concrete meaning: instantaneous rate of change. For polynomials—which form the foundation of calculus—this rate of change is visible in the algebraic structure itself. Students can see this relationship geometrically and verify it algebraically using reasoning they already possess (Tall, 2013).

This approach exemplifies structural emergence: abstract concepts arising naturally from systematic attention to concrete relationships. Students don't sacrifice mathematical sophistication—they develop conceptual understanding that supports abstraction rather than replacing it. When students later encounter formal definitions and rigorous proofs, these formalize relationships they already understand intuitively.

What This Means for Your District

The abstract emphasis in traditional calculus instruction creates artificial barriers while failing to develop the conceptual understanding that calculus applications actually require. This represents profound misalignment between instructional means and disciplinary ends.

Pedagogical Reorientation: Concrete problem contexts should provide entry points to calculus concepts, with abstraction emerging as the summary and the systematization of concrete experiences. Geometric visualization should support rather than compete with algebraic reasoning. Applications-first approaches build both motivation and conceptual foundation. The secondary developmental sequence in mathematics should mirror historical development: from concrete problems through intuitive solutions toward abstract systematization.

STEM Pipeline Implications: Accessible calculus approaches naturally connect to other STEM disciplines where calculus applications occur—physics, engineering, biology, economics, and data science. Students encounter mathematics as a tool for understanding phenomena rather than as abstract exercises disconnected from experience. This integration supports STEM persistence by helping students see mathematics as useful rather than as an arbitrary credential requirement (Rasmussen & Kwon, 2007).

The Political Economy of Abstraction: The abstraction requirement maintains calculus as scarce credential accessible primarily to students who can tolerate high levels of decontextualized symbol manipulation—a cognitive style cultivated through extensive academic preparation and correlated with socioeconomic advantage. When we challenge the necessity of abstraction-first pedagogy, we're not proposing students learn less rigorous mathematics. We're arguing that rigor should be built on understanding rather than replacing it. Twenty years of work with the accessible calculus demonstrates that students can engage sophisticated mathematical reasoning through concrete foundations.

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Misconception #5: "Calculus Is the Capstone of the High School Mathematics Sequence"

The Problem

We've organized secondary mathematics education around a powerful but limiting metaphor: calculus as capstone—the crowning achievement sitting atop a carefully constructed prerequisite edifice. This metaphor shapes everything: curriculum sequencing, course offerings, teacher preparation, student aspirations. Calculus appears as destination rather than journey, as culmination rather than foundation, as reward for the mathematically elite rather than a literacy requirement for all students.

The capstone metaphor creates perverse incentives throughout the system. It positions calculus access as a scarce resource to be rationed rather than a fundamental literacy to be developed. It makes senior-year calculus enrollment into a status marker rather than mathematical competence. It encourages racing through prerequisites to reach the capstone rather than building deep understanding. Most damagingly, it places calculus perpetually out of reach for the majority of students, who never complete the long prerequisite sequence or who stumble along the way and never recover access.

But the metaphor also obscures a more productive curricular architecture. What if calculus concepts—the mathematics of change, rate, accumulation, optimization—aren't the capstone but the core of secondary mathematics? What if these ideas, properly presented through elementary foundations, could organize and motivate the entire three-year sequence of Algebra I, Geometry, and Algebra II? What if making change explicit and central rather than implicit and deferred could enhance rather than compromise algebraic and geometric understanding?

The capstone metaphor treats calculus as occurring only after algebra and geometry when historically and conceptually, calculus ideas are about algebraic and geometric relationships. We study quadratic functions in Algebra I without revealing that the linear coefficient describes instantaneous rate of change—the derivative concept. We study area in Geometry without connecting to it accumulation functions—the integral concept. We create artificial separations between mathematical domains that could, instead, illuminate each other, all to preserve calculus as remote capstone rather than immediate foundation.

This curricular architecture reflects particular historical choices about how to organize mathematical knowledge for institutional management rather than optimal learning. The capstone structure emerged when calculus was genuinely advanced material requiring years of preparation because the prerequisite mathematics itself was poorly understood and inefficiently taught. But pedagogical innovation has changed what's possible. We now possess approaches making core calculus concepts accessible through elementary foundations—yet we maintain curricular structures designed for obsolete constraints.

We should ask directly: in whose interests does the calculus as capstone model serve? It maintains calculus as a credential rather than as competence, preserving status hierarchies that concentrate advanced mathematical access among already-privileged populations. It protects traditional course structures and textbook markets from disruption. While it may provide comfortable familiarity for teachers and administrators, it systematically excludes the majority of students from ideas that should constitute basic mathematical literacy in an increasingly quantitative world.

The Reality

The Accessible Calculus Project demonstrates that calculus concepts can function as foundational organizing principles for secondary mathematics rather than culminating achievements. When we build the three-year sequence of Algebra I, Geometry, and Algebra II around explicit attention to change, rate, and accumulation, several remarkable transformations occur.

First, algebraic and geometric content become more meaningful. Students studying quadratic functions understand why they're manipulating these expressions—not just to solve equations or graph parabolas, but to reason about rates of change, to understand how position and velocity relate, to model phenomena exhibiting quadratic relationships. The linear coefficient b in the expression $y = ax^2 + bx + c$ stops being a mysterious parameter and becomes readable

information: the instantaneous rate of change of the graph at the y -intercept. Polynomial manipulation gains a purpose beyond procedural fluency—students are reasoning about derivatives encoded in algebraic structure (Crombie & Grant, 2012).

Second, mathematical connections become visible. When calculus concepts organize the secondary mathematics curriculum, students see relationships between algebraic manipulation, geometric visualization, and quantitative reasoning about change. They recognize how different mathematical representations—symbolic, graphical, numerical, contextual—express the same underlying relationships. This connective understanding produces more robust and transferable mathematical knowledge than compartmentalized course structures (Hiebert & Carpenter, 1992).

Third, more students develop greater levels of mathematical competence. A three-year trajectory developing calculus concepts through elementary foundations gives every student time to build the understanding that traditional senior-year calculus courses compress into inadequate timeframes. Students who need extended engagement with concrete representations get that time. Students who struggle with particular topics can continue developing calculus concepts through alternative pathways rather than being permanently excluded. The result: broader participation without compromised achievement (Boaler, 2016).

This curricular reimagining doesn't eliminate limit-based calculus—it provides a foundation, making formal calculus more accessible and meaningful. Students who spend three years developing intuitive and applied understandings of derivatives, integrals, and the fundamental theorem of calculus through elementary algebraic and geometric reasoning are extraordinarily well-prepared for formal calculus with limits. They encounter formalization as the systematization of relationships they already understand rather than as arbitrary abstractions. This produces both broader access to entry-level calculus concepts and stronger preparation for advanced study.

International models demonstrate the viability of integrated approaches. The Japanese curriculum, for instance, doesn't isolate calculus as a senior-year capstone but instead weaves calculus ideas throughout secondary mathematics, helping students see calculus as a natural extension and application of algebraic and geometric thinking rather than as a separate discipline (Schmidt et al., 2005). These integrated approaches produce higher achievement across broader student populations than the American capstone model.

The key insight: calculus isn't advanced because it's difficult; it appears difficult because we present it as disconnected from foundations. When we reconstruct calculus by systematically attending to elementary algebraic and geometric relationships, it becomes accessible not as a simplified version but as genuine mathematical reasoning grounded in solid conceptual foundations. The three-year developmental trajectory transforms calculus from an elite credential accessible to few into a form of mathematical literacy achievable by many.

What This Means for Your District

Reimagining calculus as core rather than capstone requires fundamental curricular restructuring—but restructuring with profound implications for equity, achievement, and STEM preparation (while addressing key mathematics standards and practices).

Curricular Redesign:

A core calculus approach reorganizes the entire secondary mathematics sequence:

- Algebra I explicitly develops derivative concepts through polynomial analysis, making rates of change central to understanding functions rather than a marginal enrichment
- Geometry explicitly develops integral concepts through area and volume calculations, connecting geometric measurement to accumulation functions
- Algebra II extends and deepens both derivative and integral concepts while building the fundamental theorem of calculus through increasingly sophisticated applications

This isn't adding calculus on top of the existing curriculum—it's reorganizing existing content around calculus ideas as unifying themes. The algebraic manipulation, geometric reasoning, and functional understanding currently taught

remain present, but gain coherence and purpose through calculus framing. Students aren't learning more content; they're learning existing content more meaningfully.

The Three-Year Developmental Trajectory:

The core calculus approach provides time for deep understanding. Rather than compressing calculus into a single senior-year course accessible only to students who've successfully navigated every previous gate, we develop calculus competence gradually across three years. This temporal distribution has profound equity implications—students who need extended engagement with particular concepts get that time without being permanently excluded from advanced mathematics.

By the end of this three-year sequence, students possess:

- Intuitive understanding of derivatives as rates of change, accessible through algebraic and geometric reasoning
- Intuitive understanding of integrals as accumulation, accessible through an examination of the area and volume of geometric objects
- Recognition of the fundamental theorem of calculus connecting derivatives and integrals
- Preparation for limit-based formalization that makes formal calculus meaningful rather than mysterious

Implications for Limit-Based Calculus:

The core calculus approach doesn't replace Advanced Placement Calculus or college calculus—it provides a foundation, making these courses more accessible to more students. Districts can offer limit-based calculus as a fourth-year option for students seeking advanced placement credit, but now from a position of strength: students arrive having spent three years developing conceptual understanding rather than racing through prerequisites. The result: higher achievement among students taking formal calculus, and broader mathematical competence among all students, whether or not they continue to limit-based courses.

This creates differentiated pathways without tracking: all students develop core calculus literacy through the three-year sequence, while some students extend to formal calculus with limits. This differs fundamentally from current models that provide calculus concepts only to students in accelerated tracks while excluding others entirely.

STEM Pipeline Transformation:

When calculus becomes core mathematical literacy rather than an elite credential, more students develop mathematical identities, positioning them for STEM participation. They understand mathematics as a tool for reasoning about change and accumulation in physical, biological, economic, and social systems—the contexts in which STEM fields apply mathematics. This increases both STEM interest and STEM preparation, particularly among populations historically excluded from advanced mathematics (Rasmussen & Kwon, 2007).

Students who might never have enrolled in senior-year calculus because they didn't complete prerequisites—or because they perceived calculus as beyond their ability—instead develop calculus competence as a natural extension of their algebraic and geometric learning. This dramatically expands the pool of students developing the mathematical tools necessary for college STEM success.

Teacher Development Requirements:

Reimagining calculus as core requires substantial teacher preparation. Most secondary mathematics teachers have never encountered calculus grounded in elementary foundations, and many don't have a deep understanding of how calculus concepts connect to the algebra and geometry they currently teach. Teachers need sustained professional development that helps them:

- Reconstruct their own understanding of calculus through elementary foundations
- Recognize how polynomial structure encodes derivative relationships

- See how geometric measurement connects to integral concepts
- Design instruction that makes calculus ideas explicit rather than implicit throughout secondary mathematics

This professional development represents a significant investment—but an investment yielding substantial returns through enhanced mathematical literacy across entire student populations rather than merely improved outcomes for traditionally served populations.

Assessment Challenges:

Core calculus approaches require assessment instruments evaluating conceptual understanding and mathematical reasoning rather than just procedural fluency. Current standardized assessments may not capture the understanding that three-year calculus development produces. Districts need both:

- Formative assessments tracking conceptual development across three years
- Summative assessments demonstrating calculus competence built through elementary foundations
- Communication strategies explaining to colleges and other external audiences what students know and can do

The College Board's recent Advanced Placement Precalculus framework, which emphasizes reasoning about change and rate, represents a move toward recognizing these competencies—but significant assessment development work remains (College Board, 2023).

The Political Economy of the Capstone:

We should acknowledge what the capstone model accomplishes institutionally. It maintains familiar course structures and scheduling patterns. It preserves calculus as a status marker differentiating college-bound from non-college-bound students. It protects established textbook markets and teacher preparation programs. It provides a comfortable level of predictability for colleges expecting students to arrive with specific course sequences.

Challenging the capstone model threatens these institutional arrangements. It requires acknowledging that we've organized secondary mathematics around administrative convenience and status preservation rather than optimal learning. It demands a willingness to disrupt comfortable patterns in the service of broader mathematical literacy and genuine equity.

When we argue for calculus as core rather than capstone, we're proposing that mathematical education should develop competence across entire populations rather than credential elite subgroups. We're asserting that calculus ideas—reasoning about change, rate, and accumulation—are fundamental literacy in the quantitative age rather than advanced knowledge for mathematical specialists. We're claiming that current curricular architecture maintains artificial scarcity serving gatekeeping functions rather than educational purposes.

Twenty years of work in the Algebra Project with the Accessible Calculus demonstrates this reorganization is mathematically sound, pedagogically effective, and equity-enhancing. The question isn't whether calculus can function as a core rather than a capstone—evidence clearly shows it can. The question is whether we have the institutional courage to reorganize secondary mathematics around what serves student learning rather than what preserves familiar structures.

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Conclusion: An Opportunity for Transformation

These five misconceptions about calculus prerequisites are not merely pedagogical errors—they represent systemic barriers that exclude hundreds of thousands of students annually from advanced mathematics education and STEM career pathways. More troubling, these barriers operate with brutal demographic predictability, disproportionately excluding students from Black, Latino/a, Indigenous, and economically disadvantaged communities—the very populations our economy most urgently needs in mathematical and scientific fields.

The evidence presented in this paper demonstrates that much of what we consider necessary for calculus readiness is actually artificial, constructed through historical precedent and institutional convenience rather than mathematical necessity. Students can engage genuine calculus concepts through elementary algebraic and geometric foundations as early as Algebra I. They can develop sophisticated mathematical reasoning without traditional four-year prerequisite sequences. They can succeed in advanced mathematics when we design curricula for development rather than selection, for inclusion rather than exclusion.

But recognizing these misconceptions represents only the first step. Genuine transformation requires systematic change across multiple institutional domains: how we conceptualize mathematical accessibility, how we prepare teachers, how we design curriculum, and how we assess mathematical understanding. It demands that we prioritize student learning over institutional tradition, equity over gatekeeping, mathematical sense-making over procedural compliance. Most fundamentally, it requires acknowledging that current systems accomplish their exclusionary outcomes not accidentally but structurally—and that transformation therefore demands structural intervention, not merely improved implementation of existing approaches.

The districts that embrace this transformation will position themselves to develop the diverse mathematical talent that our increasingly quantitative economy desperately needs. Those school systems that maintain traditional gatekeeping structures will continue wasting human potential while failing to prepare students—all students—for participation in the mathematical culture that increasingly shapes economic opportunity, civic engagement, and scientific literacy (National Research Council, 2013; Moses & Cobb, 2001).

The Equity Imperative: Beyond Individual Access to Systemic Change:

Mathematical accessibility isn't merely an educational issue—in the 21st century it constitutes a fundamental matter of social justice that determines who has opportunity to participate in an increasingly mathematical economy and society. When we deny students access to mathematical knowledge that controls economic and social opportunities, we perpetuate systemic inequities that undermine both democratic participation and economic mobility. Mathematics functions as gatekeeper to STEM fields, to quantitative literacy necessary for informed citizenship, and to career pathways offering economic security. Excluding students from advanced mathematics is thus excluding them from full social participation (Martin, 2009; Gutstein, 2006).

But here's the deeper issue most districts haven't confronted directly: these inequities are largely *artificial*. They persist not because mathematics is inherently exclusive or because only certain populations possess mathematical capacity, but because we've normalized institutional arrangements that serve gatekeeping functions. We've mistaken historically contingent curricular choices for mathematical necessities. We've confused cognitive stratification with academic standards. The accessible calculus approaches documented in this report offer concrete pathways for dismantling these artificial barriers while maintaining—indeed, enhancing—mathematical rigor and conceptual depth.

The equity imperative operates at multiple scales simultaneously. At the individual level, accessible calculus creates opportunities for students who would otherwise be excluded from advanced mathematics, opening STEM pathways that current systems foreclose. At the institutional level, it challenges deficit narratives about mathematical ability that rationalize exclusionary practices. At the systemic level, it disrupts the reproduction of educational inequality that concentrates mathematical access among already-privileged populations. Transformation at any one level without addressing the others produces limited, fragile change. Genuine equity requires coordinated intervention across all three scales.

The Implementation Challenge: From Recognition to Action

Recognizing these misconceptions is a necessary but insufficient condition for change. The real challenge lies in systematic implementation that can navigate institutional resistance, overcome resource constraints, address legitimate concerns about rigor and college preparation, and maintain focus on student learning and equity outcomes throughout the inevitably difficult transition period.

After twenty years of working with districts across diverse geographic, demographic, and resource contexts, we've learned that successful transformation requires far more than good intentions or inspiring rhetoric. It demands strategic professional development that reconstructs teachers' mathematical understanding. It requires curriculum resources aligned with accessible approaches—resources that currently exist outside mainstream textbook adoption cycles. It necessitates assessment tools that evaluate mathematical reasoning rather than procedural compliance—tools that may not align perfectly with standardized testing regimes. And it requires administrative structures that support rather than constrain mathematical accessibility, which often means challenging scheduling practices, placement policies, and tracking systems that currently organize secondary mathematics education (Oakes et al., 1997).

The implementation challenge is simultaneously technical and political. Technically, it involves developing new curriculum materials, assessment instruments, and pedagogical strategies. Politically, it involves confronting stakeholders who benefit from current arrangements, addressing parental anxieties about college admissions, and navigating institutional cultures that resist change. Districts that attempt transformation without attending to both dimensions typically fail—either producing technically sound innovations that generate overwhelming political resistance, or politically palatable initiatives that lack technical coherence to produce meaningful change.

Beyond Professional Development: The Deep Work of Institutional Change

The districts willing to implement accessible calculus share common characteristics. They're willing to question fundamental assumptions about mathematical ability and who possesses it. They prioritize student learning over institutional tradition and administrative convenience. They approach equity as a mathematical issue rather than merely a political concern, recognizing that expanding access requires curricular transformation, not just changed placement policies. And they commit to sustained engagement rather than episodic intervention (Elmore, 2004).

This deep work cannot be accomplished through isolated workshops, inspirational conference presentations, or purchased curriculum programs. It requires sustained engagement with both the mathematical foundations of accessible approaches—understanding how calculus concepts can be reconstructed through elementary tools—and the pedagogical strategies necessary for classroom implementation. Most importantly, it demands institutional courage to challenge systems that currently exclude, even when those systems appear natural or essential to stakeholders invested in current arrangements.

The professional development required differs fundamentally from typical district offerings. Rather than procedural training in new techniques, teachers need opportunities to reconstruct their own mathematical understanding—to discover how calculus concepts emerge from algebraic and geometric foundations, to experience derivative thinking through polynomial analysis, to work through integral calculations using geometric reasoning. This intellectual work, undertaken by teachers as mathematical learners, generates both content knowledge and pedagogical vision that mere technique training cannot provide (Ball & Cohen, 1999).

The Professional Development Your District Needs

Mathematical accessibility requires a deep understanding of both the theoretical foundations and the practical implementation of advanced mathematics, approached through elementary means. District leaders ready to transform their mathematics programs need intensive professional development addressing:

1. The Mathematical Foundations of Accessible Calculus

Understanding how calculus concepts can be approached through elementary algebraic and geometric tools while maintaining—indeed, enhancing—mathematical rigor and conceptual depth. This includes:

- How polynomial structure encodes derivative relationships accessible in Algebra I
- How geometric transformation and area calculation provide integral foundations in Geometry

- How the Fundamental Theorem emerges naturally from boundary analysis implicit in Cavalieri's Principle
- How formal limit theory can be approached as a systematization of relationships students already understand concretely

2. Classroom Implementation Strategies

Developing pedagogical approaches that support accessible mathematics while addressing legitimate concerns about academic standards, college preparation, and educational quality. This includes:

- Literacy-based instructional strategies that help students read mathematical structure
- Experiential lesson design that positions students as mathematical sense-makers
- Formative assessment practices that evaluate reasoning rather than mere procedure
- Discourse strategies that support mathematical argumentation and collaborative knowledge-building

3. Systemic Change Management

Navigating institutional resistance, engaging community stakeholders, and developing policy frameworks that support rather than constrain mathematical accessibility. This includes:

- Strategies for addressing parental concerns about college admissions and academic rigor
- Approaches for working with counselors and administrators whose placement practices currently restrict access
- Methods for communicating with colleges about non-traditional pathways
- Frameworks for documenting student learning in ways that demonstrate achievement to external audiences

4. Assessment and Evaluation

Creating measurement systems that evaluate mathematical reasoning and conceptual understanding rather than procedural compliance alone. This includes:

- Performance assessment design that captures mathematical thinking
- Rubric development aligned with reasoning rather than answer-getting
- Portfolio approaches that document mathematical growth over time
- Strategies for supplementing standardized assessments with locally-developed instruments that better capture the understanding accessible approaches develop

5. Teacher Preparation and Support

Building departmental capacity for approaches that may challenge traditional mathematical presentations while enhancing student understanding and expanding access. This includes:

- Professional learning community structures that support ongoing teacher learning
- Mentoring and coaching models that provide classroom-embedded support
- Curriculum development processes that engage teachers as designers rather than mere implementers
- Strategies for cultivating mathematical leadership within departments

Your Next Steps: From Recognition to Implementation

District leadership now faces a defining choice: maintain prerequisite structures that systematically exclude students from advanced mathematics, or implement pathways designed for systematic inclusion. The mathematics curriculum is field-tested—built on over thirty years of Algebra Project development (Moses, Kamii, Swap, & Howard, 1989), with the polynomial calculus implementations first documented in Ypsilanti and Mansfield classrooms (Grant et al., 2012) and currently being refined through the NSF-funded Accessible Calculus Project in New Jersey and Georgia school districts. The pedagogical methods are validated—tested across varied demographic and institutional contexts through classroom-embedded professional learning approaches. What remains is a leadership decision: whether to ground district mathematics policy in empirical evidence and developmental research, or to preserve institutional arrangements that serve administrative convenience rather than educational equity.

For District Supervisors Ready for Transformation

If these misconceptions resonate with challenges you're observing in your district's mathematics program—if you're seeing talented students systematically excluded from advanced mathematics through artificial barriers—if you're

concerned about equity gaps that persist despite good intentions and significant resource investment—if you recognize that current approaches aren't producing the broad-based mathematical competence your community needs—then you're ready for the deeper work of systemic change.

The Algebra Project offers intensive professional development specifically designed for district leadership teams ready to implement accessible calculus approaches that expand access while maintaining rigor. Our programs address both the mathematical foundations and practical implementation challenges that determine whether accessibility initiatives succeed or fail, whether they produce genuine transformation or merely symbolic gestures toward equity.

Professional Development Programs Available

5-Day Intensive Workshop: The Mathematics of the Accessible Calculus

This comprehensive workshop explores how to develop calculus concepts using foundational mathematics accessible in Algebra I, Geometry, and Algebra II. Participants engage calculus as learners, discovering how derivative and integral concepts emerge from elementary algebraic and geometric reasoning. The workshop addresses both mathematical content—understanding polynomial structure, geometric transformation, boundary analysis—and pedagogical strategy—how to support students in constructing these insights themselves. Participants leave with deep content knowledge, curriculum resources, and implementation strategies for integrating calculus concepts throughout secondary mathematics.

5-Day Implementation Workshop: Classroom Strategies for Accessible Mathematics

This practical workshop equips educators with effective pedagogical techniques for accessible mathematics instruction. Focus areas include literacy-based activities that help students read mathematical structure, experiential lesson design that positions students as active sense-makers, formative assessment strategies that evaluate reasoning, and discourse practices that support mathematical argumentation. Participants work through complete lesson sequences, analyze student thinking, and develop implementation plans for their specific contexts. The workshop addresses common challenges—pacing pressures, mixed-readiness classrooms, standardized testing requirements—with practical strategies grounded in successful implementations.

Design Team Development and Execution

This sustained engagement program involves periodic consultations with district leadership teams, providing ongoing support for managing change toward district-wide implementation. The program focuses on developing site-based design teams responsible for overseeing systemic transition, fostering cross-departmental collaboration, engaging community stakeholders, and driving sustainable reform. Design teams receive coaching on change management, curriculum development, professional learning facilitation, and assessment design. This multi-year engagement produces both immediate classroom impact and long-term institutional capacity for continued innovation.

Follow-up Professional Learning and Coaching Support

Recognizing that sustainable transformation requires ongoing support beyond initial training, the Algebra Project provides flexible hybrid coaching and professional learning options for districts implementing accessible calculus approaches. Following the intensive 5-day workshops, participating educators and leadership teams can access monthly virtual coaching sessions, quarterly on-site classroom visits, and asynchronous support through a dedicated online learning community. This follow-up support addresses implementation challenges as they arise, provides accountability structures that maintain momentum, and creates opportunities for educators to refine their practice through collaborative reflection. The hybrid model combines the depth of in-person coaching with the accessibility and responsiveness of virtual support, ensuring that districts receive the sustained guidance necessary to move from initial implementation to systemic integration. This ongoing partnership typically extends 12–18 months following initial workshops, with intensity customized to district needs and implementation progress.

These programs are specifically designed for district supervisors, curriculum coordinators, department chairs, and teacher leaders who are ready to question fundamental assumptions about mathematical accessibility while maintaining an unwavering commitment to academic excellence and intellectual rigor.

The Investment Your Students Deserve

Mathematical accessibility isn't a luxury—it's a necessity for any district serious about equity, college readiness, and STEM participation in the 21st century. The cost of maintaining current exclusionary systems—measured in wasted human potential, lost economic productivity, and perpetuated social stratification—far exceeds the investment required for transformation.

Consider: every student excluded from calculus through artificial prerequisites represents unrealized potential. Multiply that individual loss by the thousands of students affected annually in even medium-sized districts, then compound it across years and across the hundreds of districts that maintain these barriers. The aggregate human cost is staggering. And it falls most heavily on communities that have already borne disproportionate educational injustice (Ladson-Billings, 2006).

The investment in transformation—professional development, curriculum resources, assessment tools, implementation support—appears modest against this backdrop. And unlike many educational interventions that show small, fragile effects, accessible calculus approaches produce dramatic, durable expansion in participation in advanced mathematics (Burris et al., 2006). The return on investment, measured in terms of student outcomes, substantially exceeds that of typical professional development initiatives.

Ready To Begin?

Contact The Algebra Project to discuss how accessible calculus approaches can transform your district's mathematics program while addressing persistent equity gaps that resist conventional interventions. Professional development programs are available throughout the 2025-2026 academic year and summer 2026, with scheduling flexible to accommodate district calendars and specific implementation timelines.

The students in your district already possess the mathematical talent necessary for advanced study—all of them, not just the narrow demographic slice currently accessing calculus. The only question is whether we'll create systems that develop that talent or continue maintaining barriers that systematically waste it. That choice is yours. The mathematics, the pedagogy, the evidence, and the support systems are all ready. What's needed now is institutional courage to act on what we know rather than what's comfortable, to prioritize what is just rather than what is familiar.

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THE FIVE REALITIES

- 01.** "Calculus Grows From Basic Algebra and Geometry, Not After Years of Prerequisites"
- 02.** "Limits Formalize Calculus Understanding, They Don't Create It"
- 03.** "Calculus Competence Develops Through Opportunity and Effort, Not Giftedness"
- 04.** "Calculus Builds Abstract Mathematical Thinking"
- 05.** "Calculus Should Be the Core of High School Mathematics, Not the Capstone"

William (Bill) Crombie is Director of Professional Development for the Algebra Project and Principal Investigator of the Accessible Calculus Project. His research focuses on removing artificial barriers to advanced mathematics while maintaining mathematical rigor and enhancing student understanding.

At the Algebra Project, we believe that in the 21st century every child has a civil right to secure math literacy – the ability to read, write and reason with the symbol systems of mathematics.

The Algebra Project is a 501c3 registered non-profit organization working with teachers, schools, and communities to ensure students who have not previously had access to a college preparatory sequence can make the conceptual shift from arithmetic to algebraic thinking. Our primary focus is on building collaborations with middle schools and high schools to ensure that students are successful in higher-level mathematics, attaining an on-time high school graduation in which they are fully prepared for college or career without need for remediation in math.

Get in touch today to learn more about our work!

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