Five-Step Process: Step One
By Aidan Soguero & Bill Crombie

In Algebra Project classrooms, the first step to learning mathematics is to capture student engagement. And so, the first step to the Five-Step Curricular Process is a shared physical event, a popular example being a trip. Students will hop on a bus or subway or simply do a walking tour together, and travel through town, marking landmarks along the way.

This isn’t done as some bait-and-switch whereupon returning to the classroom they’re suddenly asked to figure out what X equals in relation to the trip. It really is just a trip. Something they can discuss using their own natural language, what we call “people talk,” when they return. And it is done as a necessary on-ramp to students viewing math as accessible and approachable.

Because of the strict focus on experience, this may give some the false impression that the experience itself is remedial, or “not really math.” Meanwhile, viewing math only as the rote memorization of equations is, what others may argue, “not really math.” These shared experiences are valuable for camaraderie and onboarding, but they also are mathematically rich. Having such experiences enables students to interpret the abstract equations of mathematics from a more concrete perspective. This, in turn, allows them to become more effective learners of mathematics.

Bill Crombie, Director of Professional Development at the Algebra Project, sat down with Greg Budzban, a research mathematician active in curriculum development with the Algebra Project, to discuss how mathematics actually emerges from physical experience. Or, in his words, “Just like collisions in physics produce all sorts of new particles, I think the collision of consciousness with the physical world somehow creates ideas.” (Cont’d on page 3)
COMMUNITY VOICES

Introverted Activism: How becoming a Math Literacy Worker Showed one Student the Importance of Her Voice
by Aidan Soguero

Anna Njie never struggled much with the content of her classes. An accomplished cellist, she passed an Advanced Placement Calculus class with no teacher and no other classmates, and is now on the premed track at Vanderbilt. So, while she may seem an autodidact and star student, there remained an arena where she never felt fully comfortable, and that was socially. When the COVID-19 pandemic shut down in-person learning, her social anxiety truly came to a head.

“When everything went virtual, that was even more nerve-wracking. I couldn’t just raise my hand to speak. I had to go and unmute the mic and that was just a whole lot of anxiety for me.” she said, speaking about her first summer as a Math Literacy Worker with the Young People’s Project (YPP). “Going into that first summer of teaching YPP modules, I wasn’t really at the forefront, I would just kind of listen.”

Anna attended Martha Ellen Stilwell School of the Arts in Clayton County, Georgia, throughout middle school and high school. She was introduced to the Young People’s Project at 16, after focusing primarily on music for her school career. Having never struggled with math, but also having no particular affection for it, she decided to become a Math Literacy Worker simply to try something different.

“I got introduced through a mass email the district sent out about a math teaching internship. And at this point, I was sick of music. So I was like, let me try this out. So yeah, I applied, we got trained, and that was my first introduction to it. But at that point I was like, I don’t know if this is for me because I was really shy and I didn’t like to speak up about anything.”

Math Literacy Workers (MLWs) are a peer-to-peer math initiative started by the Young People’s Project to train students with Algebra Project pedagogy who then get paid to do math with other students who may be struggling. By doing math alongside other students, there is a better rapport from the onset which is linked to overcoming mathphobia, and by paying MLWs for their time it instills a culture of valuing the labor of students.

“I did a lot of listening and learning that year about … teaching and being an MLW, and then during the next summer, I was more comfortable and I was like, ‘you know what? These people paid me to do this and I didn’t even do anything last time. So maybe I should actually participate and try and teach this time.’ It brought me out of my comfort zone and taught me that, a lot of times, what I have to say is important.”

From there she formed a relationship with the Baltimore Algebra Project (BAP) and began joining the National Math Literacy Corps (NMLC) meetings they host. The Tuesday night NMLC meetings hosted and run by youth but with an adult mentor, Jay Gillen, featured youth from across the country.

“I was one of two people joining from Atlanta. Most people were from Baltimore, and they had experience with the organizing side of things and, like, bringing people together for a common cause and whatnot.” Conversation soon turned to the possibility of a town hall.

They began with a mock town hall in Tacoma, just the size of a classroom, but it was a big stepping stone for Anna. “I was kind of leading it, which I hadn’t done before.”

From there, they moved on to the real, and much larger, town hall, “It was on July 12th, during the Free Minds, Free People Conference. It was pretty successful. Not only did I relay the same message that I presented at Tacoma, but it was to a whole different audience. And these were math people and people that support youth doing things, and support the idea of math literacy being a set goal for everyone, something that should happen.”

Jay reached out to let her know of an opportunity with the
emerging We the People – Math Literacy for All Alliance, and the Algebra Project, which were collaborating on a national conference for July 2022. Organizers felt youth representatives were critical for the planning of the conference. Not only did he recommend that she be part of the organizing committee, he suggested that she and fellow Math Literacy Corps member Jamaria Hall host the opening plenary session. But Anna was still a little hesitant in groups, and especially in groups of strangers.

“Jay mentioned that there was a working group starting to plan this conference. And he said that if I was interested in joining a meeting, I could, and I was a little intimidated because being in a room of people that I don't know is still scary to me.” Anna did attend a planning meeting, but quickly decided it wasn’t for her.

Jay, however, was persistent. “(By the end of the school year) Jay was talking to me and he kind of puts these ideas in my head that he knows I'll agree with. I just need to hear someone else say it. And so he was saying that they need youth voice on the working group team and kind of encouraging me to be a recurring member on those calls. So I agreed. And I was telling him about my needing a job for the summer to save for college, and he was like, ‘well, why don't we try and get you paid for this?’” Anna was funded with support from the Center for Innovation in STEM Education Research (CiSTEMer) at Kennesaw State and BAP.

Anna and Jamaria became paid student voices on the planning call and worked together to present the opening plenary session of the conference. Their session was the most well-attended presentation out of the over 40 groups presenting. You can watch all of the conference recordings, including Anna and Jamaria’s, here.

Patience in teaching her new things, allowing her to acclimate at her own speed, and being paid for her time and labor all contributed to Anna becoming the organizer she is today. And she hopes to help other young people get the same resources she did as she heads off to college.

“I'm going to study Medicine, Health and Society, I'm on the premed track. But I'm also opening up myself to teaching professionally. And I really do hope to, you know, still be involved in the Math Literacy Corp, and even the things that are happening in my school district. Because I think that's what made me put myself out there to apply to schools I wouldn't think I'd get into. And so I don't wanna leave them behind. I want that to be something that I grow with and not grow away from.”

(Cont'd from page 1) The following interview has been edited for length and clarity...

**Bill:** Can you tell us a little bit about your history with the Algebra Project, your work with Bob Moses?

**Greg:** I began as a research mathematician who did work in, you could call it probabilistic decision processes.

Then I met Bob and we did some work together on developing curriculum for the Algebra Project. And as I'll mention in a moment, it was really his conception of the Five-Step Curricular Process and what I thought about where mathematics emerges, and there was this very nice synergy between his ideas and mine.

That really got me excited in a lot of different ways about working with him and the Algebra Project. Later, I went into university administration, became chair of the math department at Southern Illinois University, Carbondale. And then eventually Dean of Arts and Sciences at the SIU Edwardsville campus. Two years ago, I retired from that. And I've been working with the Project on various things ever since.

**Bill:** I would like to kick off with a very broad question, from your perspective, where does mathematics come from?

**Greg:** I started off in pure mathematics. I also did work on the applied side basically because I needed to pay the bills. But, being on both sides of that line certainly, I think, reinforced my philosophical ideas about where mathematics comes from and that it does, I think, emerge from experience. The way I've described it is, what we call consciousness collides with what we call the physical world. And from that collision, all sorts of things happen. Just like collisions in physics produce all sorts of new particles, I think the collision of consciousness with the
physical world somehow creates ideas.

I'm not enough of a metaphysician or philosopher – I call myself an amateur philosopher – but I'm certainly not well read enough in metaphysics to dig deep into that idea.

I think that one of the fundamental problems that Eugene Wigner points out in his famous paper is the unreasonable effectiveness of mathematics in describing the world. Something that seemingly is so pure and, in fact, so pure that Plato stuck it in this ideal world of forms. It was seemingly that pure. But using mathematics, physics can produce just amazingly accurate and precise predictions of the world and models the world in a very, very precise way. How can something so ideal actually fit the world in the way it does? And the thing that makes sense to me is that mathematics emerges from the world from physical experience.

And so (in spite of Wigner’s concern over unreasonableness) it’s not necessarily a confusion. You can turn around and actually model experience. Because if mathematics emerges from experience from the beginning, then the fact that you can model the physical world and experiences within the physical world using mathematics makes sense.

Think of the Five-Step Process like a circle. You start with the experience, you represent it, you do people talk, you do feature talk and you do symbolic representations. And then we talk about the applications of those symbolic representation. So now you circle back through the Five-Step Process, and do it all again, but this time applying what you have created about the shared experience.

Bill: Okay, so in other words, you start with the (1) shared experience, you represent it with (2) drawings or intuitive models, you write (3) “people talk” descriptions of the experience, and then you (4) identify key features - create “feature talk”, which then you can (5) represent with symbols, doing symbolic representations. You know, the Algebra Project and the Five-Step Curricular Process are focused on achieving math literacy within students most underserved by the system. Can you speak to the pedagogical and educational implications about this idea of where math comes from, and the importance of this First Step, especially as it relates to the population the Algebra Project is most concerned with?

Greg: Yes. I think the Five-Step Process works so well as a pedagogical structure because it is faithful to where the mathematics comes from. How we develop mathematics is starting from experience. So if that's how we develop mathematics, starting from experience, why wouldn’t we want to teach it that way? Unfortunately the problem is that because these experiences lead to abstraction, and the abstractions are a very effective and efficient way at encapsulating the mathematics, a traditional mathematics classroom likes to start with the abstractions.

Bill: Could you give a specific example, maybe with the Road Coloring Unit you helped develop? What abstraction is that getting at?

(Example of Road Coloring Problem as both a “built” city and as a diagram)

Greg: For those who aren't familiar with the Road Coloring Problem, it was a problem that emerged in the 1960s in symbolic dynamics, in theoretical computer science. It was this
idea you can describe colloquially as a collection of buildings with one-way roads leading away from them. This network of buildings and roads had to satisfy certain properties. The two properties were that every building has exactly two one-way roads leaving away from them. This network of buildings also had a property that no matter what building you start in, there’s a sequence of roads that allowed you to get everywhere in the so-called city and back to where you started.

I won’t go into the set of research problems that emerged from that. But trying to find a way to name the roads so that you can find an instruction, a common instruction, that will get everybody in the city that you built to the same place at the same time was a very difficult problem.

It remained open for a long period of time and that was what I was studying at the time when I met Bob Moses originally. One way of thinking about that problem is when the students are physically doing that problem and literally walking around this city, following the roads and following the instructions, they’re having a physical experience of what we call a mathematical function. And functions are arguably the most important idea in mathematics. I mean, you eventually learn in math that basically everything is a function. Everything important is a function.

And Bob was able to see that students could take this problem and the students could physically build these cities. When I met him, I was working on the problem as a research problem, but I never physically built a city. I would draw circles and arrows on a page. Or I would use various representations, like matrices. But I never physically built a city. I mean, it was Bob's idea to actually have the students physically build these things. and play with them with little markers and coins. The fact that he was able to see that the students could physically do this, that there was a real physical experience that the students could have, was part of his genius. So, when I saw that, it was a transformational moment for me. It was like, “oh my gosh, this is what he means by the Five-Step Process.”

And it was clear, the students were gaining intuition into this idea of a function.

There’s this notion of these experiences being longitudinal, there’s the ability to both provide students who are struggling with mathematics with a physical and intuitive idea and experience, but also take students who are later in their mathematical careers and still give them things which are both interesting and have a great deal of mathematical content.

Bill: So, what you’re saying is, the Five-Step Curricular Process, and your example of the Road Coloring Unit, are not a remedial device. It’s a rich mathematical experience accessible at multiple levels.

Greg: There’s a depth to the ideas that emerge that, while they provide an accessible way into the ideas, there isn’t a ceiling in terms of the mathematics ideas. When you start students off with this physical intuition and accelerate them through the process they can access ideas that are very deep.

So, it’s not remediation. It’s not doing the same thing over and over again and expecting different results. I mean, that’s the problem with remediation, right? In some sense, it’s the very definition of pedagogical insanity.

But this is really giving students a completely different experience. Once they get engaged, you can literally accelerate them through the learning progression.

Bill: You’ve developed curriculum that is mathematically rich and accessible at multiple levels, but can you speak to your work with teachers, teaching them this curriculum and the professional development work you do?

Greg: That’s an important aspect to this. In some sense, I think teachers at the middle school and high school level face the same issues that students do. That is to say that the standard curriculum in many ways obscures the mathematical ideas. For many teachers, I think it has become a collection of rote procedures, and not having a chance to play, literally play, with the ideas.

So, teachers themselves are looking for new ways to engage students. They know the obstacles that some of the traditional mechanisms, the traditional pedagogy, represent for many of their students. Especially teachers who are dealing with underrepresented students that oftentimes have difficulty with the abstract. Those teachers are even more desperate. So what is most gratifying for me is to see the teachers themselves
become excited and passionate and almost re-energized in their profession when they have experiences with the Five-Step Process and the Algebra Project curriculum and pedagogy.

That’s important because students sense when a teacher is passionate. Students sense when a teacher is energetic and interested in the ideas that they themselves are teaching. Seeing teachers being re-energized and themselves rethinking things about, “okay, well, this is an interesting way of handling functions, there are some ideas in this I need to excavate.” The fact that teachers are learners side-by-side with students in Algebra Project classrooms, is also part of the power of the Five-Step Process. We’re all learning mathematics.

I make this point, that the amount of mathematics that I don’t know is most of it, so when I teach an upper-level class, a graduate student class, if I don’t get a question that I literally don’t know the answer to, I feel like I haven’t really done my job.

It’s important to have teachers become comfortable with the idea that they are learners of mathematics, and they don’t have to act like they’re the experts that know all the answers. The best research mathematicians in the world don't know all the answers, don’t know all the mathematics. That’s an important part of the professional development working with the Five-Step Process with teachers and re-engaging them in the ideas. It’s providing ways for them to see themselves as fellow learners with the students, but also giving them ways to engage and excite the students about the subject.

Bill: Finally, how would you define math literacy? When do we know when students are math literate?

Greg: That’s a great question. I think there’s a collection of ideas that they have to become effective at understanding. But it’s almost as important for them to re-imagine themselves as learners of mathematics. In other words, this is a subject that they can learn. When you’re literate in an area, it means that could pick up something and you can engage with it. It’s accessible to you.

I say that I can read because I can pick up a book and I can actually make sense of it. So I think it’s learning a collection of ideas and perhaps more importantly, that we get students to the place where they see themselves as learners of mathematics. In other words, it’s a subject where they’re empowered enough and confident enough that they can pick up some mathematics and not be daunted by it. Have it be a subject that really is accessible to them. Changing that mindset is at least, or even more, important than any one set of procedures or ideas. We give students the confidence that they are truly mathematically literate and are learners of mathematics.

**NEWS**

**The We the People – Math Literacy for All Alliance** and the Algebra Project, Inc. Co-sponsored a 2022 online national conference in July. Session recordings will be made available on algebra.org this Fall.

**The Algebra Project is beginning work with** Confluence Academies in St. Louis, Missouri schools, as well as Plainfield Public Schools in New Jersey as part of an overall effort to provide more resources to teachers and students throughout the country.

**Broward County Public Schools**, Florida, hosted a Summer Induction and Professional Development workshop with the Algebra Project to welcome students and teachers back to school this past July.

**The Bob Moses Speaker Series Conference** will be held at MIT January 28th and 29th, 2023. For more information, please visit www.bobmosesconference.com

**Algebra Project Board member**, Professor Margaret A. Burnham has recently published a new book, *By Hands Now Known - Jim Crow's Legal Executioners*. In conjunction, the Burnham-Nobles Digital Archive, chronicling 1,000 cases of lynchings of Black folks in the South between 1930-1954 and book release will be the subject of an October 7th conference at Northeastern University. Register here.
The Young People’s Project will be hosting a gala to celebrate their 25th anniversary, October 1st, 2022. The Algebra Project will be in attendance and hosting its own pod where we will be play Trip Cards and reviewing AP resources. You can find more information about the event on YPPs website, www.typp.org/anniversary
With gratitude to...

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We thank our many contributors and are grateful for your continued support as we steward Bob Moses’ vision of raising the floor of mathematics literacy for all children in America! Please contact us c/o ben@algebra.org or call 1-617-491-0200 with any questions.

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